

Residential Real Estate Investment: Optimal Holding Period with Taxation

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Presentation schedule

- Taxation issues;
- Optimal holding period of a real estate asset;
- Optimal holding period with taxation;
- Concluding remarks.

- High complexity: Taxation depends on the kind of investment (financial assets, real estate assets, gold...);
- It depends on wealth levels, of other incomes (for example labor income...);
- For standard decision criteria such as the expected utility, usually the weight invested on the risky asset is decreasing with taxation if we consider only the investment on the financial market. However, taking account of the whole investor's income, it depends on parameters (taxation rate, relative risk aversion).

- *Tax Optimization* (pay less taxes legally) versus *Portfolio Optimization*; (France's example)
- For financial assets: "Livret d'Epargne", "Plan d'Epargne en Actions", "Assurance-Vie"...And for real estate investment, successive and various tax exemption laws: 2003: Robien's law; 2006-2010: Borloo's law; January 2009: Scellier's law instead of Robien's law; January 2013: Duflot's law instead of Scellier's law; September 2014: Pinel's law instead of Duflot's law (1st September 2014 to 31st December 2016, potentially prolonged), and other laws such as Censi Bouvard's law (January 2013-December 2017), the Malraux's law still active ! *To be continued...*).
- Taxation depends also on the length of the management period. It is crucial to determine the capital gains for real estate investment (see what follows).

Optimal holding period of a real estate asset

- Usually, real estate investment is viewed as a rather passive process (buy-and-hold strategy ensuring relatively stable cash flows for many years) avoiding too large transaction costs.
- Determination of the optimal time to sell a real estate. Holding period is defined as the expected period of time during which an investment is or should be kept.
- Many theoretical and empirical research studies deal with this problem (see Atkins and Dyl, 1997; In, Kim and Gençayc, 2011 and Lim and Kim, 2011). For the USA case, Hendershott and Ling (1984) and Gau and Wang (1994) emphasize tax laws condition holding durations.
- Rowley et al. (1998) show that holding period decision is linked to depreciation and obsolescence. For commercial real estate, Collett et al. (2003) observe that the median holding period is about seven years (UK database). See also Brown and Geurts (2005).

Optimal holding period of a real estate asset

Denote k the discount factor (for corporate investment, usually the weighted average cost of capital (WACC)). The free cash flow grows at a constant rate g .

The real estate asset price dynamics follows a geometric Brownian motion:

$$\frac{d\tilde{P}_t}{\tilde{P}_t} = \mu dt + \sigma dW_t, \quad (1)$$

where W_t is a standard Brownian motion where parameters μ and σ are respectively the trend and volatility.

The sum of the discounted free cash flows FCF_s is equal to:

$$C_t = \int_0^t FCF_s e^{-ks} ds = \int_0^t FCF_0 e^{-[k-g]s} ds, \quad (2)$$

The real estate portfolio value V is defined as the sum of the discounted free cash flows (FCF) and the discounted terminal value (the selling price):

$$V_t = C_t + P_t. \quad (3)$$

Optimal holding period of a real estate asset

Determination of T^* : For any time t in $[0, \bar{T}]$, the investor has to choose between selling the asset or not.

- If the price return μ is higher than the WACC k , then, the optimal solution is simply equal to the maturity \bar{T} . Thus, in what follows, we consider the case $\mu < k$.

We also focus on the subcase $g < \mu$, (see empirical data)

Consequently, not selling the asset implies a higher sum of the discounted free cash flows (C_t is increasing with t) but a smaller discounted expected terminal value $P_0 e^{[\mu-k]t}$.

- The optimization investor problem is: (maximization of the expectation)

$$\text{Max}_{t \in [0, \bar{T}]} \mathbb{E} [V_t]. \quad (4)$$

Optimal holding period of a real estate asset

Case 1: The initial price P_0 is smaller than $\frac{FCF_0}{k-\mu} e^{-(k-\mu)\bar{T}}$.

Then, the optimal time to sell T^* corresponds to the maturity \bar{T} . Since the Price Earning Ratio (PER) $\frac{P_0}{FCF_0}$ is too small ($< \frac{e^{-(k-\mu)\bar{T}}}{k-\mu}$), the sell is not relevant before maturity.

Case 2: The initial price P_0 lies between the two values $\frac{FCF_0}{k-\mu} e^{-(k-\mu)\bar{T}}$ and $\frac{FCF_0}{k-\mu}$.

Then, the optimal time to sell T^* is given by:

$$T^* = \frac{1}{\mu - g} \ln \left(\frac{FCF_0}{P_0} \times \frac{1}{k - \mu} \right). \quad (5)$$

Case 3: The initial price P_0 is higher than $\frac{FCF_0}{k-\mu}$.

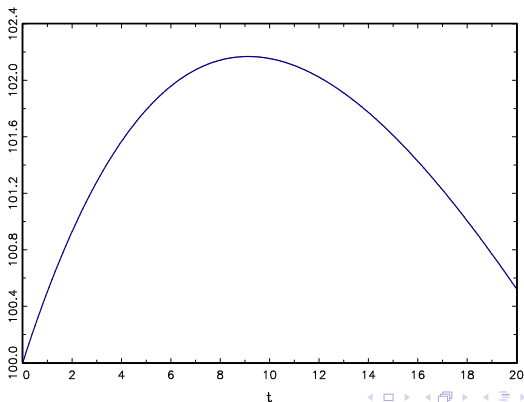
Then, the optimal time to sell T^* corresponds to the initial time 0. Since the PER $\frac{P_0}{FCF_0}$ is sufficiently large ($> \frac{1}{k-\mu}$), there is no reason to keep the asset P .

Optimal holding period of a real estate asset

$$\bar{T} = 20, \mu = 4.4\%, g = 3\%, k = 8.4\%, P_0 = 100, FCF_0 = 100/22.$$

The discounted expected value V_t of the portfolio is concave and achieves its maximum at $T^* = 9.13$.

V_t



Optimal holding period with taxation

- Numerical base case for taxation: The tax τ_F on the cash flows is equal to 30%. The property tax τ_P applied to the cash flows is equal to 8%. The tax τ_G on the capital gains is equal to 30%. The acquisition fee f_a when buying initially the asset is equal to 5%. Finally the costs of work $f_c(.)$ is equal to 15%.
- Two kinds of degression function:
 - The first one (Germany, Spain and England) corresponds to a piecewise constant functions, namely $lc(.)$ is equal to 0 on the time interval and equal to 1 for maturity longer than 10 years (it means that the taxation on the capital gains is null after ten years);
 - The second one, which a simplified version of the French case, corresponds to a piecewise linear functions, namely $li(.)$ is equal to a piecewise linear function on the time interval (it means that the taxation on the capital gains is linearly decreasing on the first 22 years) and equal to 1 for maturity longer than 22 years.

Optimal holding period with taxation

The optimization problem is:

$$\text{Max}_{t \in [0, \bar{T}]} E[V_t].$$

with

$$V_t = C_t + P_t - \tau_G P_0 \cdot \text{Max} \left[e^{[(\mu - 0.5\sigma^2)t + \sigma W_t]} - (1 + f_a + f_{cw}(t)), 0 \right] (1 - l(t)) e^{-k_P t}.$$

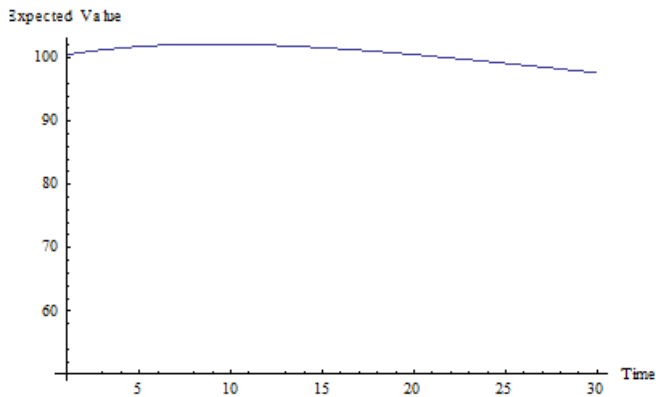
$$P_t = P_0 \exp \left[\left(\mu - k_P - \frac{1}{2} \sigma^2 \right) t + \sigma W_t \right] \text{ with } E \left[\frac{P_t}{P_0} \right] = \exp \left[(\mu - k_P) t \right]$$

$$C_t = \int_0^t FCF_0 e^{-[k_F - (g - \tau_F - \tau_P)]s} ds = \frac{FCF_0}{k_F + \tau_F + \tau_P - g} \left(1 - e^{-[k_F + \tau_F + \tau_P - g]t} \right)$$

Optimal holding period with taxation

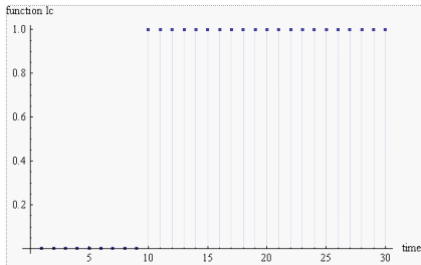
Numerical case (without taxation)

$$\mu = 4.4\%, \sigma = 5\%, g = 3\%, k_F = k_P = 8.4\%, P_0 = 100, FCF_0 = 100 / 22$$

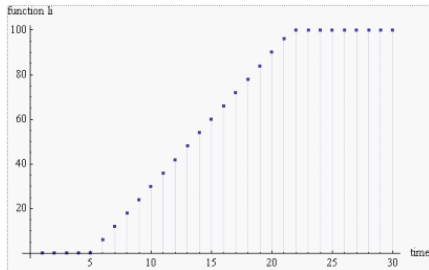


The optimal time to sell is $T^* = 9$ years.

Optimal holding period with taxation

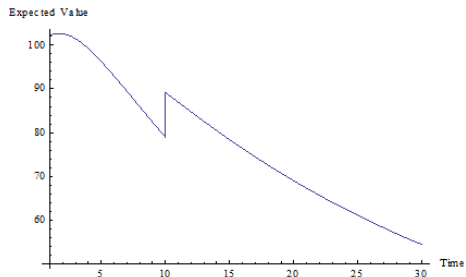


Degression function lc

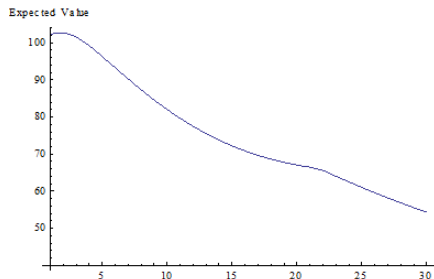


**Degression function li "
(simplified" French case)**

Optimal holding period with taxation



Expected value (degression Ic)

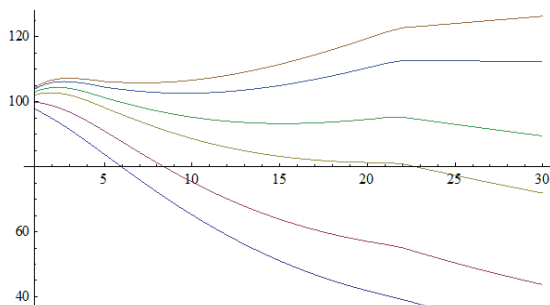


Expected value (degression li)

Optimal holding period with taxation

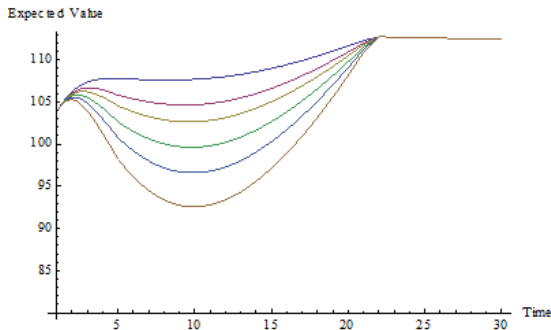
- Sensitivity to the trend of the real asset price: Both the expected value and the optimal time to sell increase with the trend (according to intuition).

The optimal time to sell the real estate asset can suddenly jump for example from about 3 to about 10 years (see for instance the blue curve corresponding to 8.35%). For the higher trend value, it would be optimal to sell at the upper bound on the maturity, namely 30 years for this example.



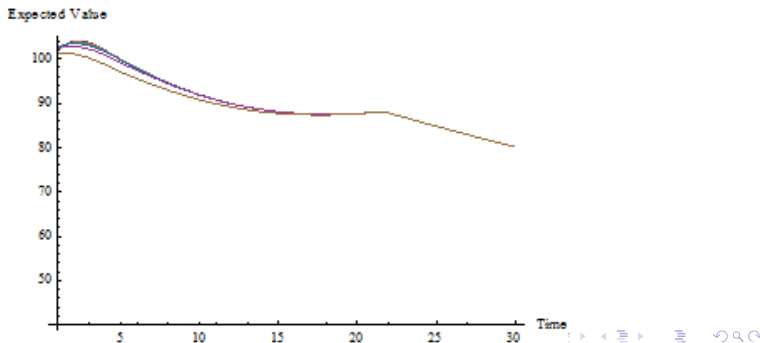
Optimal holding period with taxation

Sensitivity to the tax on the capital gains. We investigate six cases for τ_G : 15%, 24%, 30%, 48% and 60%. The expected value is obviously decreasing with respect to the tax on the capital gains. If the time horizon is smaller than 10 years, the optimal time to sell is (slightly) decreasing with respect to tax when the trend of the real estate asset is not too high.



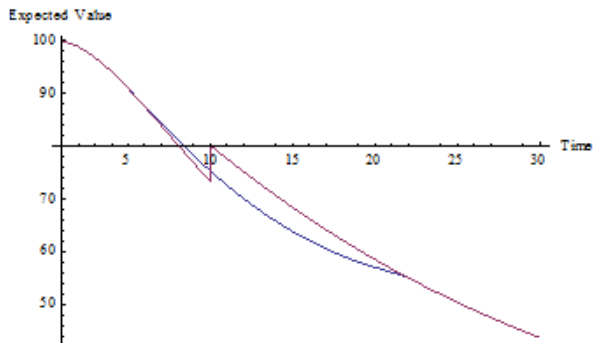
Optimal holding period with taxation

- Sensitivity to the real estate asset volatility: The expectation of the portfolio value decreases with respect to the volatility. But the impact of the volatility is not very significant for small trend values and/or standard real estate volatility levels. Recall that, for the risk-neutrality case, the volatility does not play any role when taxation is not taken into account.



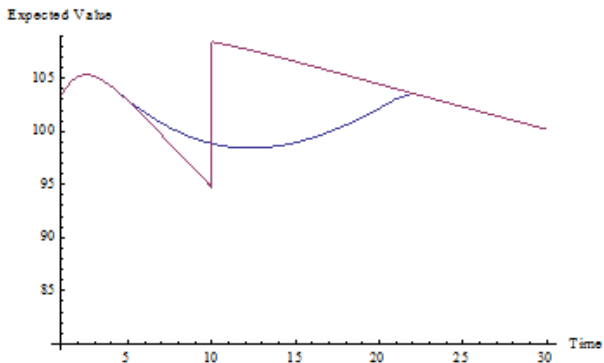
Comparison of the li case (France) and the lc case (England, Deutschland, Spain)

trend=2.2%



Comparison of the li case (France) and the lc case (England, Deutschland, Spain)

trend=8%



Optimal holding period with taxation

- As expected, the difference is significant for high values of the trend. In such a case, the difference between the two times at which there is no longer taxation on the capital gains has a strong influence on the expectation of the global wealth.
- For this numerical example, when the trend=8%, we note that, if the terminal date is smaller than 10 years, the optimal holding period are equal but, for longer maturities, they highly differ.
- Indeed, for the lc case, the optimal holding period is equal to the time at which the taxation on the capital gains does no longer hold (here 10 years) while, for the li case, it corresponds to an early selling (about 3 years) and not at all to the time at which there is no more taxation on the capital gains (here 22 years).

Concluding remarks

- Taxation on the real estate market impacts the optimal holding period very significantly in particular for high trends of the real estate asset.
- Introducing two standard degressive taxations on the capital gains, we show that the corresponding optimal times to sell may highly differ.
- Note also that, despite the continuity of the expected value with respect to time for one of these digressive functions, the optimal time to sell the real estate asset can suddenly jump.

Concluding remarks and extensions

- If the fund is not closed (i.e. the optimal date is not computed once for all at the initial time), use the American option approach (see Barthélémy and Prigent, 2009).
- Take account of the real estate volatility: risk-aversion as illustrated by Amédée-Manesme et al. (2016) when there is no taxation.
- Examine the role of the financing of property (here the property was already paid, or we were in the Modigliani-Miller framework as regards the active/passive management).
- Incorporate lease structure effect in order to better account for the specificities of real estate (see Amédée-Manesme et al., 2015);
- All processes of interest are in fact stochastic (but only simulations could be used).
- Compare several types of holding periods including in particular the buy-and hold investment strategy (see Amédée-Manesme et al. (2016) using compensating variation).