

# Hedging Labour Income Risk over the Life-Cycle

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## Motivation: Theory vs. Measurement

Human wealth is similar to a risky asset, due to income shocks

$$\text{Total Wealth} = \text{Financial Wealth} + \text{Human Wealth}$$

→ Individual saving and investment decisions depend on labour income

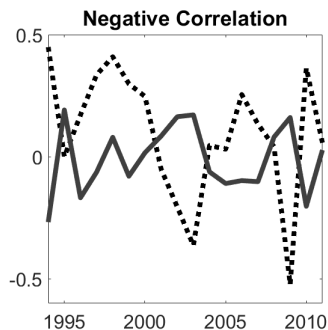
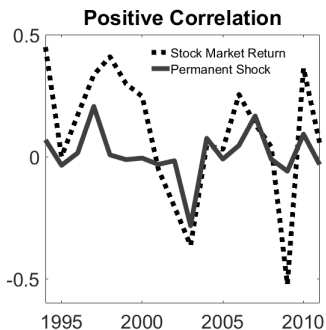
- A risk-averse saver may use stocks to reduce total wealth risk, hedging **permanent income shocks (PIS)**  
Constantinides and Duffie 1996, Heaton and Lucas 1996, Viceira 2001, Haliassos and Michaelides (2003), Angerer and Lam (2009)
- A saver should reduce her equity investment (Merton, 1969), and even stay out of the stock market (Bagliano et al. 2014) **only if her PIS display positive correlation with stock returns**
- Savers may **update their estimates of correlation, based on realized PIS** if there is learning about labour income (Chang et al.(2018))

→ Ex-ante and revised correlation should explain both stock market participation and asset allocation across cash, bonds, stocks.

## Motivation: Income Shocks-Equity Return Correlation

Positive correlation  $\Rightarrow$  Stocks amplify income shocks

Negative correlation  $\Rightarrow$  Stocks help hedging income shocks



## Motivation: Theory vs. Measurement

Measuring the income-hedging motive is challenging because of challenges in estimating unobserved correlation

1. Correlations should differ across individuals: large N
  2. Limited time-series dimension of the data
    - => estimates are imprecise and may not differ from zero
  3. PIS shocks are not observable. Decompose stochastic income growth into temporary and permanent components
    - based on structural models. This requires a long time series of income data  
Carroll (1992, 1997); Meghir and Pistaferri (2004)
    - PIS in  $t$  is the equal weighted average of the stochastic income growth rates in years  $(t - 1)$ ,  $t$  and  $(t + 1)$ 
      - The transitory component at year  $t$  is the difference between the year  $t$  stochastic labor income growth and the permanent component of income growth at year  $t$ .
- => limited economic significance of correlation

## Contribution

- We extend the specification of the labour income process due to **Cocco Gomes Maenhout (2005)** (CGM), to allow for individual (or clustered) correlations
  - A Minimum Distance (MD) estimation of correlations, based on the restrictions of the CGM income process, exploits the large cross-sectional data dimension
  - Clustered MD correlations matter for both participation and asset allocation, also in economic terms
- **Bonaparte et al.(2014)** (BKK) use sample (realized) individual correlations, explaining both participation choice and average frequency
  - We reconstruct the dynamics of PIS through a Kalman Filter, and recursively update correlation estimates
  - These revised correlation estimates explain the revised participation choice better than our MD estimates of correlation

# This Presentation

## 1. Method

## 2. Empirical Analysis

- Data
- Results- with BKK as benchmark

Individual and Clustered MD Correlations

Explaining Participation and Asset Allocation

Reconstructing the Dynamics of PIS

Explaining Revised Participation with Revised Correlations

# 1. Method

- Labour income is the sum of three components (as in CGM): a deterministic one, associated to personal characteristics; a transitory shock and a permanent income shock (PIS).

PIS have an aggregate and an idiosyncratic component

We assume that the aggregate component of PIS co-moves with stock returns according to the individual (or cluster) correlation.

▶ Individual Correlations

- This implies that the PIS of any two savers co-move with each other, only because each moves with stock return.  
==> We exploit the cross-sectional dimension of the data:  $N(N-1)/2$  restrictions on a function of income shocks (DRES), allowing for the Minimum Distance estimation of  $N$  correlation coefficients  
==> MD correlations and MD80 correlation
- We then use MD estimates and realized returns to measure PIS and recursively reconstruct the idiosyncratic PIS from DRES.

==> KF correlations

▶ Moment Conditions

## 2. Data

Dutch Household Survey (DHS) (Bonaparte et al. (BKK)[JFE, 2014])

- Panel Data of Households (1993-2011), N=1456
- Personal Characteristics
- Investment Decisions

Variable	Mean	Standard Deviation	p10	Median	p90	N (n x T)
OwnSTK	0.12	0.32	0	0	1	14999
OwnMF	0.18	0.38	0	0	1	14999
OwnSTKMF	0.27	0.44	0	0	1	16976
Ln(NetWorth)	10.90	2.07	8.89	12.17	13.08	10116
Ln(NetIncome)	9.75	0.841	8.76	9.94	10.51	13694
Corr(d(LnInc),Rm)	-0.07	-0.41	-0.60	-0.08	0.50	27664
SD(d(LnInc))	0.39	0.49	0.06	0.21	0.96	27664
HH size	2.45	1.19	1	2	4	16945
Age	53.9	13.70	35	54	72	16945
Education	0.564	0.496	0	0	1	16945
Male	0.613	0.487	0	1	1	16945
Unemployed	0.117	0.322	0	0	1	16945
Retired	0.284	0.451	0	0	1	16945
Health	3.91	0.693	3	4	5	14809
RiskAversion	4.44	2.05	1	5	7	13833



# Clusters

$\rho_i$ , the ex-ante individual correlation parameter, may depend on ex-ante observable characteristics

Similar individuals should have similar correlations

⇒ We repeat the estimation of ex ante correlations on homogeneous **clusters** of individuals

$$\rho_i \Rightarrow \rho_j$$

Clustering variables

- are available for most waves
- forecast individual PIS
- are stable over time (i.e. Risk aversion, Gender, Education..)

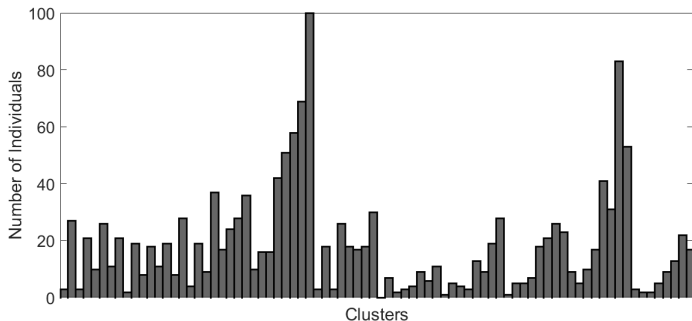
**Advantages:**

- **Econometric:** higher conditions/parameter ratio
- **Economic:** easier design of initial portfolios for new investors

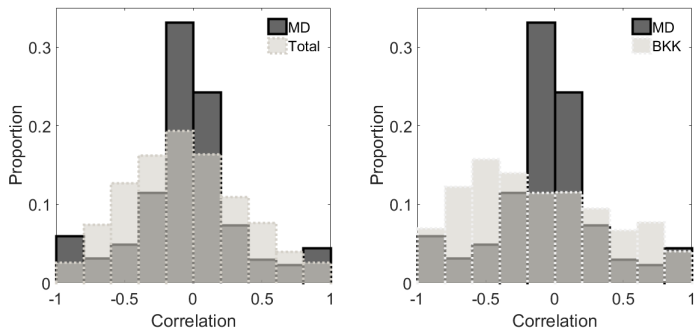
## Cluster Population

80 Clusters: Gender (2), Education (5), Urbanization (2), Financial Literacy (2), Risk Aversion (2)

**Distribution of Population Across Clusters**



## 2.1 Results: Sample vs. MD (left) and BKK (right)

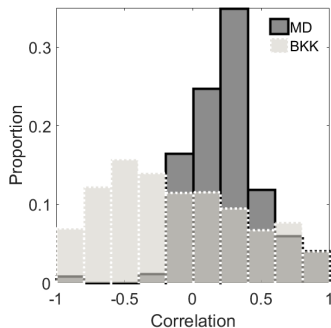
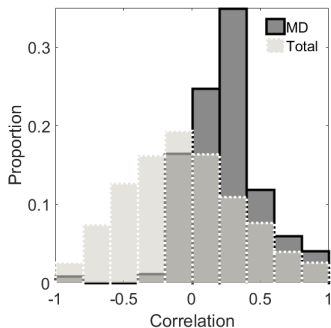


### Correlation between PIS and stock returns

- is more concentrated around the mean, being driven by stock returns;
- has thicker tails, because there is no orthogonal temporary shocks as in both total and BKK-averaged income shocks.

## Sample vs. MD-Clustered (left) and BKK (right)

### Clustered Ex-Ante Correlation Estimates (MD80)



## 2.2 Explaining Stock Market Participation and Asset Allocation

$P_{i,t} = 1$ , if individual  $i$  owns stocks at time  $t$

$W_{i,t}$  = share of individual's financial wealth invested in stocks

We explain  $P_{i,t}$  with a Probit regression and  $W_{i,t}$  with a Tobit regression, changing the component of labor income growth used to define the correlation with returns

- total labor income growths:  $\text{Corr}(\text{tot}, \text{Rm})$
- deterministic component of labor income growth:  $\text{Corr}(\text{det}, \text{Rm})$
- stochastic labor income growth (PIS and transitory):  $\text{Corr}(\text{stoc}, \text{Rm})$
- based on moving average transitory component:  $\text{Corr}(\text{transBKK}, \text{Rm})$
- based on moving average PIS:  $\text{Corr}(\text{permBKK}, \text{Rm})$
- based on Clustered MD PIS:  $\text{Corr}(\text{permMDA80}, \text{Rm})$
- based on MD PIS:  $\text{Corr}(\text{permMDAll}, \text{Rm})$

Relevance

- a. strength of the income hedging motive
- b. structural models vs. moving average for estimating PIS and correlation: does it make a difference? (DeBacker et al. 2013)

## Stock Market Participation

Dummy variable  $P_{i,t} = 1$ , if individual  $i$  owns stocks at time  $t$

→ Explaining  $P_{i,t}$  with Probit regression onto

- Total income growth volatility St.Dev (dy)
- Income-Return Correlation (Income-Hedging motive)
- Personal Characteristics (Age, Gender, Education, Employment,...)

	OwnSTKMF
Independent Variable	
St. Dev. (dy)	
Corr(tot,Rm)	
Corr(det,Rm)	
Corr(stoc,Rm)	
Corr(transBKK,Rm)	
Corr(permBKK,Rm)	
Corr(permMDA80,Rm)	
Corr(permMDAll,Rm)	
N	
Pseudo $R^2$	

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Dummy variable  $P_{i,t} = 1$ , if individual  $i$  owns stocks at time  $t$

→ Explaining  $P_{i,t}$  with Probit regression onto

- Total income growth volatility St.Dev (dy)
- Income-Return Correlation (Income-Hedging motive)
- Personal Characteristics (Age, Gender, Education, Employment,...)

OwnSTKMF	
Independent Variable	
St. Dev. (dy)	<b>-0.092</b>
Corr(tot,Rm)	<b>-0.280</b>
Corr(det,Rm)	
Corr(stoc,Rm)	
Corr(transBKK,Rm)	
Corr(permBKK,Rm)	
Corr(permMDA80,Rm)	
Corr(permMDAll,Rm)	
N	7,585
Pseudo R <sup>2</sup>	0.259

## Stock Market Participation

Dummy variable  $P_{i,t} = 1$ , if individual  $i$  owns stocks at time  $t$

→ Explaining  $P_{i,t}$  with Probit regression onto

- Total income growth volatility St.Dev (dy)
- Income-Return Correlation (Income-Hedging motive)
- Personal Characteristics (Age, Gender, Education, Employment,...)

OwnSTKMF	
Independent Variable	
St. Dev. (dy)	<b>-0.141</b>
Corr(tot,Rm)	
Corr(det,Rm)	0.107
Corr(stoc,Rm)	
Corr(transBKK,Rm)	
Corr(permBKK,Rm)	
Corr(permMDA80,Rm)	
Corr(permMDAll,Rm)	
N	8,501
Pseudo R <sup>2</sup>	0.256



## Stock Market Participation

Dummy variable  $P_{i,t} = 1$ , if individual  $i$  owns stocks at time  $t$

→ Explaining  $P_{i,t}$  with Probit regression onto

- Total income growth volatility St.Dev (dy)
- Income-Return Correlation (Income-Hedging motive)
- Personal Characteristics (Age, Gender, Education, Employment,...)

Independent Variable	OwnSTKMF
St. Dev. (dy)	<b>-0.088</b>
Corr(tot,Rm)	
Corr(det,Rm)	
Corr(stoc,Rm)	<b>-0.319</b>
Corr(transBKK,Rm)	
Corr(permBKK,Rm)	
Corr(permMDA80,Rm)	
Corr(permMDAll,Rm)	
N	8,240
Pseudo R <sup>2</sup>	0.259

## Stock Market Participation

Dummy variable  $P_{i,t} = 1$ , if individual  $i$  owns stocks at time  $t$

→ Explaining  $P_{i,t}$  with Probit regression onto

- Total income growth volatility St.Dev (dy)
- Income-Return Correlation (Income-Hedging motive)
- Personal Characteristics (Age, Gender, Education, Employment,...)

	OwnSTKMF
Independent Variable	
St. Dev. (dy)	<b>-0.092</b>
Corr(tot,Rm)	
Corr(det,Rm)	0.097
Corr(stoc,Rm)	<b>-0.316</b>
Corr(transBKK,Rm)	
Corr(permBKK,Rm)	
Corr(permMDA80,Rm)	
Corr(permMDAll,Rm)	
N	8,240
Pseudo R <sup>2</sup>	0.259

## Stock Market Participation

Dummy variable  $P_{i,t} = 1$ , if individual  $i$  owns stocks at time  $t$

→ Explaining  $P_{i,t}$  with Probit regression onto

- Total income growth volatility St.Dev (dy)
- Income-Return Correlation (Income-Hedging motive)
- Personal Characteristics (Age, Gender, Education, Employment,...)

	OwnSTKMF
Independent Variable	
St. Dev. (dy)	<b>-0.183</b>
Corr(tot,Rm)	
Corr(det,Rm)	
Corr(stoc,Rm)	
Corr(transBKK,Rm)	-0.055
Corr(permBKK,Rm)	
Corr(permMDA80,Rm)	
Corr(permMDAll,Rm)	
N	5,968
Pseudo R <sup>2</sup>	0.260

## Stock Market Participation

Dummy variable  $P_{i,t} = 1$ , if individual  $i$  owns stocks at time  $t$

→ Explaining  $P_{i,t}$  with Probit regression onto

- Total income growth volatility St.Dev (dy)
- Income-Return Correlation (Income-Hedging motive)
- Personal Characteristics (Age, Gender, Education, Employment,...)

	OwnSTKMF
Independent Variable	
St. Dev. (dy)	<b>-0.173</b>
Corr(tot,Rm)	
Corr(det,Rm)	
Corr(stoc,Rm)	
Corr(transBKK,Rm)	
Corr(permBKK,Rm)	<b>-0.084</b>
Corr(permMDA80,Rm)	
Corr(permMDAll,Rm)	
N	5,968
Pseudo R <sup>2</sup>	0.261

## Stock Market Participation

Dummy variable  $P_{i,t} = 1$ , if individual  $i$  owns stocks at time  $t$

→ Explaining  $P_{i,t}$  with Probit regression onto

- Total income growth volatility St.Dev (dy)
- Income-Return Correlation (Income-Hedging motive)
- Personal Characteristics (Age, Gender, Education, Employment,...)

OwnSTKMF	
Independent Variable	
St. Dev. (dy)	<b>-0.172</b>
Corr(tot,Rm)	
Corr(det,Rm)	
Corr(stoc,Rm)	
Corr(transBKK,Rm)	-0.073
Corr(permBKK,Rm)	<b>-0.094</b>
Corr(permMDA80,Rm)	
Corr(permMDAll,Rm)	
N	5,968
Pseudo R <sup>2</sup>	0.262

## Stock Market Participation

Independent Variable	OwnSTKMF						
St. Dev. (dy)	<b>-0.092</b>	<b>-0.141</b>	<b>-0.088</b>	<b>-0.092</b>	<b>-0.183</b>	<b>-0.173</b>	<b>-0.172</b>
Corr(tot,Rm)	<b>-0.280</b>						
Corr(det,Rm)		0.107		0.097			
Corr(stoc,Rm)			<b>-0.319</b>	<b>-0.316</b>			
Corr(transBKK,Rm)					-0.055		-0.073
Corr(permBKK,Rm)						<b>-0.084</b>	<b>-0.094</b>
Corr(permMDA80,Rm)							
Corr(permMDAll,Rm)							
N	7,585	8,501	8,240	8,240	5,968	5,968	5,968
Pseudo R <sup>2</sup>	0.259	0.256	0.259	0.259	0.260	0.261	0.262

$$P(Y_{i,t}(\rho_i = \rho_L)) - P(Y_{i,t}(\rho_i = \rho_H)) = 5\%$$

$$\rho_L = -0.6; \rho_H = +0.5$$

i.e. a switch from 10th to 90th percentile of the correlation distribution

## Stock Market Participation

		OwnSTKMF						
Independent Variable								
St. Dev. (dy)	<b>-0.092</b>	<b>-0.141</b>	<b>-0.088</b>	<b>-0.092</b>	<b>-0.187</b>	<b>-0.187</b>	<b>-0.176</b>	
Corr(tot,Rm)	<b>-0.280</b>							
Corr(det,Rm)		0.107		0.097				
Corr(stoc,Rm)			<b>-0.319</b>	<b>-0.316</b>				
Corr(transBKK,Rm)					-0.032			-0.049
Corr(permBKK,Rm)						<b>-0.108</b>	<b>-0.113</b>	
Corr(permMDA80,Rm)								
Corr(permMDAll,Rm)					<b>-0.135</b>	<b>-0.173</b>	<b>-0.163</b>	
N	7,585	8,501	8,240	8,240	5,677	5,677	7,745	
Pseudo R <sup>2</sup>	0.259	0.256	0.259	0.259	0.258	0.265	0.265	

$$P(Y_{i,t}(\rho_i = \rho_L)) - P(Y_{i,t}(\rho_i = \rho_H)) = 7\%$$

## Stock Market Participation

	OwnSTKMF						
Independent Variable							
St. Dev. (dy)	<b>-0.092</b>	<b>-0.141</b>	<b>-0.088</b>	<b>-0.092</b>	<b>-0.202</b>	<b>-0.191</b>	<b>-0.148</b>
Corr(tot,Rm)	<b>-0.280</b>						
Corr(det,Rm)		0.107		0.097			
Corr(stoc,Rm)			<b>-0.319</b>	<b>-0.316</b>			
Corr(transBKK,Rm)					-0.064		-0.081
Corr(permBKK,Rm)						-0.079	<b>-0.089</b>
Corr(permMDA80,Rm)					<b>-0.336</b>	<b>-0.326</b>	<b>-0.337</b>
Corr(permMDAll,Rm)							
N	7,585	8,501	8,240	8,240	5,677	5,677	5,677
Pseudo R <sup>2</sup>	0.259	0.256	0.259	0.259	0.264	0.263	0.264

$$P(Y_{i,t}(\rho_i = \rho_L)) - P(Y_{i,t}(\rho_i = \rho_H)) = 15\%$$



## Asset Allocation

$W_{i,t}$  = share of individual's financial wealth invested in stocks

### → Tobit Regression

Independent Variable	PropSTK								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
St. Dev. (dy)		0.019	0.076	0.076	0.093	0.050	0.087	0.051	0.093
Corr(tot,Rm)	<b>-0.092</b>	<b>-0.094</b>	<b>-0.049</b>						
Corr(det,Rm)				0.034					
Corr(stoc,Rm)				<b>-0.065</b>					
Corr(transBKK,Rm)					<b>0.057</b>		<b>0.051</b>		<b>0.057</b>
Corr(permBKK,Rm)					-0.006		-0.004		-0.005
Corr(permMDA80,Rm)						<b>-0.107</b>	<b>-0.128</b>		
Corr(permMDAll,Rm)								0.028	0.001
Corr(permKF,Rm)									
Controls	NO	NO	YES	YES	YES	YES	YES	YES	YES
N	13,283	11,371	7,434	7,431	5,577	7,581	5,577	7,581	5,577
Pseudo R <sup>2</sup>	0.003	0.003	0.282	0.283	0.310	0.279	0.311	0.311	0.311

$$W_{i,t}(\rho_i = \rho_L) - W(Y_{i,t}(\rho_i = \rho_H)) \approx 10\%$$

⇒ Income hedging motives have a sizable impact on both stock market participation and asset allocation

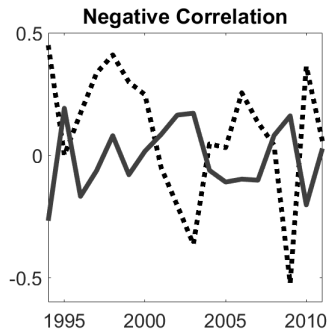
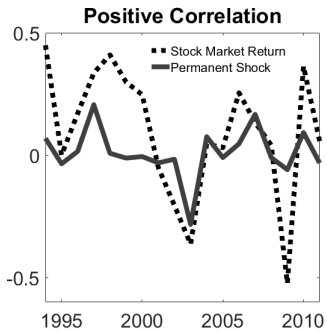
## 2.3 Reconstructing the Individual Dynamics of PIS

- Uncertainty and learning about labour income rationalize the age profile of stock investments (Chang et al.(2018)).
- Savers may update their estimates of correlation, based on realized PIS, if there is learning

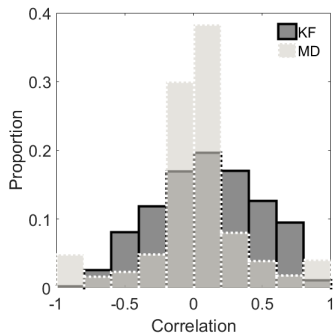
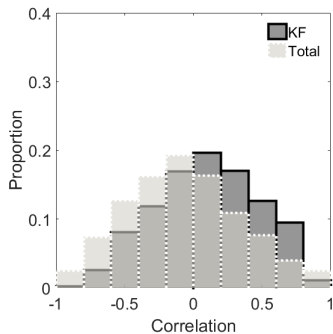
→ We exploit the co-movements with stock market to infer the PIS,  $u_{i,t}$

▶ Kalman Filter

## Realized Permanent Shocks



## Revised Individual Correlations



→ More dispersed compared to one obtained by MD

(KF based on sample realization of ex-ante correlation)

## 2.5 Explaining Participation Frequency

We now consider how the participation choice evolves over time for the same individual  
 Counting variable  $Z_i$  = number of waves individual  $i$  owns stocks

### → Poisson Regression

Independent Variable	NperiodsSTKMF								
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
St. Dev. (dy)	-0.050	-0.052	<b>-0.130</b>	<b>-0.079</b>	<b>-0.134</b>	<b>-0.076</b>	<b>-0.128</b>	<b>-0.052</b>	<b>-0.080</b>
Corr(tot,Rm)	<b>-0.171</b>								
Corr(det,Rm)		<b>0.255</b>							
Corr(stoc,Rm)		<b>-0.182</b>							
Corr(transBKK,Rm)			-0.038		-0.041		-0.036		-0.021
Corr(permBKK,Rm)			<b>-0.070</b>		<b>-0.069</b>		<b>-0.071</b>		<b>-0.095</b>
Corr(permMDA80,Rm)				-0.031	-0.049				
Corr(permMDAll,Rm)						-0.015	-0.011		
Corr(permKF,Rm)								<b>-0.195</b>	<b>-0.269</b>
N	1,147	1,147	730	1,199	730	1,199	730	1,199	730
Pseudo R <sup>2</sup>	0.252	0.257	0.278	0.248	0.278	0.248	0.278	0.251	0.283

→ What matters for the revised decision to stay in the market is the **updated correlation** between PIS and returns

## Conclusions

This paper improves the measurement of income hedging needs

showing their statistical and economic significance

supporting implications of life cycle theory with learning about uncertain income

- new estimation method exploiting both theoretical restrictions and the larger cross-sectional dimension of data
- three alternative measures of correlations between PIS and stock returns
- one measure better explains stock market participation, asset allocation and participation frequency

### Applications

- Improved design of portfolios/ target date funds

Link portfolio shares of a new investor to the ex ante characteristics of the cluster she belongs to

Revising her portfolio shares after measuring her realized permanent income shocks

## Income Process with Individual Correlations

$\log$ -Labour income  $\log(Y_{i,t})$  is a deterministic function of observable characteristics (age, sex, education, etc..) plus a stochastic component

$$\log(Y_{i,t}) = f(t, Z_{i,t}) + e_{i,t},$$

that is given by a **transitory component** and a **random walk**

$$\log(Y_{i,t}) = f(t, Z_{i,t}) + \epsilon_{i,t} + v_{i,t-1} + u_{i,t},$$

with shocks  $u_{i,t}$ , the PIS, given by **aggregate** and **idiosyncratic** components

$$\log(Y_{i,t}) = f(t, Z_{i,t}) + \epsilon_{i,t} + v_{i,t-1} + \xi_{i,t} + \omega_{i,t},$$

where  $\text{corr}(u_{i,t}, r_t) = \text{corr}(r_t, \xi_{i,t}) = \rho_i$

$$\xi_{i,t} = \sigma_u \rho_i (r_t / \sigma_r) \sim \mathcal{N}(0, \sigma_u^2 \rho_i^2); \omega_{i,t} \sim \mathcal{N}(0, \sigma_u^2 (1 - \rho_i^2))$$

$$\Rightarrow u_{i,t} \sim \mathcal{N}(0, \sigma_u^2)$$

▶ Choleski Decomposition

This simplifies to the CGM labour income process if, for all  $i$ ,

$$\rho_i = \rho.$$

▶ Back

## Choleski Decomposition

The stock market return is proportional to an aggregate shock

$$r_t = \sigma_r W_t$$

Permanent and transitory shocks are given by

$$u_{i,t} = \sigma_u W_{i,t}^P$$

$$\epsilon_{i,t} = \sigma_\epsilon W_{i,t}^q$$

The relationship between shocks is given by

$$\begin{bmatrix} W_t \\ W_{i,t}^P \\ W_{i,t}^q \end{bmatrix} = Chol \left( \begin{bmatrix} 1 & \rho_i & 0 \\ \rho_i & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \right) \begin{bmatrix} B_t \\ B1_{i,t} \\ B2_{i,t} \end{bmatrix}$$



## 1.1 Model-Implied Moment Conditions

The labour income process implies that the shocks to labour income of any two savers co-move with each other, only because each moves with stock return.

→  $[N \times N]$  symmetric matrix (covariances across individuals' shocks)

$$\begin{bmatrix} C(1,1) & C(2,1) & C(3,1) & \dots & C(N,1) \\ C(2,1) & C(2,2) & \cdot & \cdot & \cdot \\ C(3,1) & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot \\ C(N,1) & \cdot & \cdot & \cdot & C(N,N) \end{bmatrix}$$

- Co-movements across individuals' shocks are due to aggregate shocks, only

⇒ Off-diag entries informative on  $\{\rho_i\}_{i=1}^N$

$$\begin{cases} C(i,j) = \sigma_u \sigma_r \rho_i \rho_j & i \neq j \\ C(i,j) = \sigma_u^2 + 2\sigma_\epsilon^2 & i = j \end{cases}$$

⇓

$N$  parameters

$(N(N-1))/2$  conditions

(Over)-Identified model

## 1.2 Minimum Distance Estimation

We estimate

- 1 the **deterministic component** of labour income through a Panel Regression of income on Age, Sex, Education

$$\log(\hat{Y}_{i,t}) = \hat{f}(t, Z_{i,t}),$$

- 2 the **stochastic component** as the Regression Residuals

$$\log(Y_{i,t}) = \hat{f}(t, Z_{i,t}) + \hat{e}_{i,t},$$

- 3 the **Permanent Income Shocks** as the first difference of the residuals, *dres*

$$dres_{i,t} = \Delta \hat{e}_{i,t},$$

- 4 Find value of  $\theta = \{ \{\rho_i\}_{i=1}^N, \sigma_u, \sigma_\epsilon, \sigma_r \}$  that minimizes  $Q$

$$Q(\theta) = (g_M - G_M(\theta))' I_M (g_M - G_M(\theta)),$$

where  $\{g_m(\theta)\}$  and  $\{G_m(\theta)\}_{m=1}^M$  are sample and model-implied moment conditions respectively

## Kalman Filter

The model is easily expressed in state-space form

*Space-measurement equation* (Observable)

$$e_{i,t} = v_{i,t} + \epsilon_{i,t},$$

where  $\epsilon_{i,t} \sim \mathcal{N}(0, \sigma_\epsilon^2)$

*Space-transition equation* (Unobservable)

$$v_{i,t} = v_{i,t-1} + u_{i,t},$$

$$u_{i,t} = \sigma_u (\rho_i(r_t/\sigma_r) + \eta_{i,t})$$

$$\eta_{i,t} \sim \mathcal{N}(0, \sigma_u^2(1 - \rho_i^2))$$

→ Idiosyncratic shocks

Initialize the filter with two arbitrary conditions on initial value and variance:

$$v_{i,0} \quad P_{i,0}$$

Use the state equations to estimate the one-step ahead value:

$$E_0[v_{i,1}] = v_{i,0}$$

$$E_0[P_{i,1}] = P_{i,0} + Q - Q^2$$