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Intermediated Family Contracts

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With French Summary

PROVISIONAL VERSION

Robert J. Gary-Bobo
CREST, ENSAE

Meryam Zaiem
CREST, ENSAE

Contrats familiaux intermédiés

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Résumé en français de l'article de Robert Gary-Bobo et Meryam Zaiem, CREST, ENSAE.

Résumé court

L'article étudie la possibilité et les propriétés de *contrats familiaux intermédiés*. Ces contrats sont définis comme des arrangements de famille, aboutissant au partage des revenus et de la richesse au sein d'un groupe familial, formalisant un plan inter-temporel, contingent à des événements aléatoires, comme le décès ou la maladie d'un parent. Ces contrats peuvent jouer un rôle déterminant en anticipant les successions, mais ont la particularité de faire intervenir un intermédiaire financier, à la fois banquier et assureur. Le *contrat familial intermédié* est un bouquet de contrats bilatéraux, signés entre la banque et les membres d'une famille au même moment ; c'est aussi un paquet de produits financiers et d'assurance vendus simultanément aux clients de la banque, tendant à résoudre un problème d'accord familial et de succession. Ce paquet de produits regroupe des crédits hypothécaires classiques (aux enfants), des prêts viagers hypothécaires (aux parents) ou des achats en viager par la banque elle-même, des conversions en rente viagère, de l'assurance-décès (sous une certaine forme) et des transferts implicites entre parents et enfants. Suivant les pays et les contextes réglementaires et légaux, le *contrat familial intermédié* est aussi un instrument d'optimisation fiscale. L'article étudie les propriétés d'un contrat familial optimal sous trois hypothèses-clef : l'aversion pour le risque des clients ; l'existence d'un certain degré d'altruisme des parents (à l'égard de leurs enfants) ; un fort attachement des personnes âgées à leur logement. Des simulations évaluent la profitabilité de tels contrats pour le banquier. Le contrat peut s'adresser à des personnes dont les revenus sont de niveau moyen, qui sont propriétaires de leur logement (ou détenteurs d'un certain patrimoine), et qui voudraient aider leurs enfants (ou petits-enfants) à s'établir. La contribution de l'article n'est pas de mettre en évidence l'importance potentielle du crédit viager hypothécaire pour les personnes âgées, mais de montrer qu'un intermédiaire financier, en améliorant le partage des risques au sein d'une famille, peut dégager un profit non-négligeable. Secondairement, l'analyse des contrats familiaux suggère des pistes pour l'optimisation fiscale.

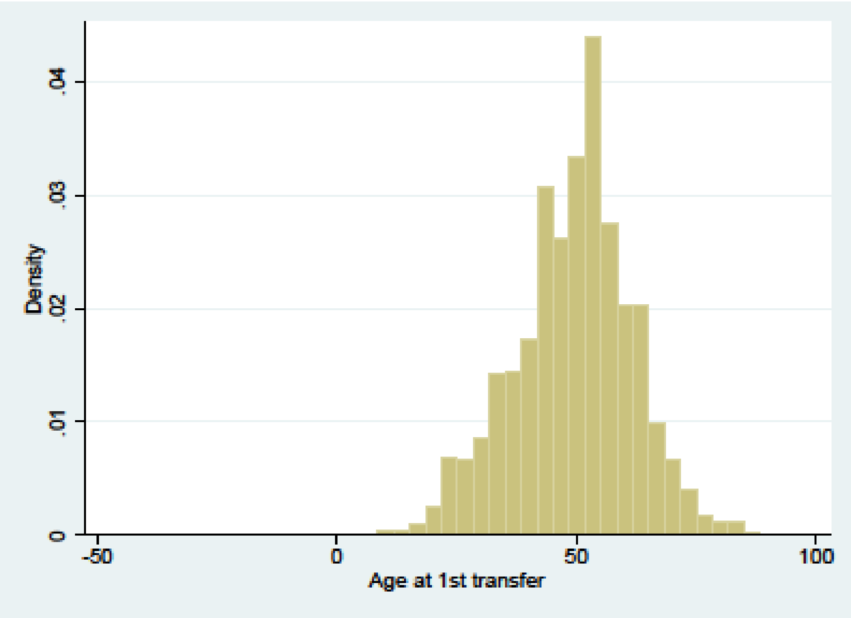
Résumé étendu

Introduction. L'article étudie la possibilité et les propriétés de *contrats familiaux intermédiés*. Ces contrats sont définis comme des arrangements de famille, aboutissant au partage des revenus et de la richesse au sein d'un groupe familial, formalisant un plan inter-temporel, contingent à des événements aléatoires, comme le décès ou la maladie d'un parent. Ces contrats ont un caractère notarial, puisqu'ils peuvent jouer un rôle déterminant en anticipant les successions, mais ont la particularité de faire intervenir un intermédiaire financier, à la fois banquier et assureur.

Le *contrat familial intermédié* est un bouquet de contrats bilatéraux, signés entre la banque et les membres d'une famille au même moment ; c'est aussi un paquet de produits financiers et d'assurance vendus simultanément aux membres de la famille cliente de la banque, tendant à résoudre un problème d'accord familial et de succession. Ce paquet de produits regroupe des crédits hypothécaires classiques (aux enfants), des prêts viagers hypothécaires (aux parents) ou des achats en viager par la banque elle-même, des conversions en rente viagère, de l'assurance-décès sous une certaine forme et des transferts implicites entre parents et enfants. Suivant les pays et les contextes réglementaires et légaux, le *contrat familial intermédié* est aussi un instrument d'optimisation fiscale. L'article étudie les propriétés d'un

contrat familial optimal sous trois hypothèses-clef : l'aversion pour le risque des clients ; l'existence d'un certain degré d'altruisme des parents (à l'égard de leurs enfants) ; un fort attachement des personnes âgées à leur logement. Des simulations évaluent le gain en bien être apporté par un contrat familial groupé optimal. Le contrat peut s'adresser à des personnes dont les revenus sont de niveau moyen, qui sont propriétaires de leur logement (ou détenteurs d'un certain patrimoine), et qui voudraient aider leurs enfants (ou petits-enfants) à s'établir. La contribution de l'article n'est pas de mettre en évidence l'importance potentielle du crédit viager hypothécaire pour les personnes âgées, mais de souligner qu'un intermédiaire financier, en améliorant le partage des risques au sein d'une famille, *peut offrir une valeur ajoutée non-négligeable à ses clients*. Secondairement, l'analyse des contrats familiaux suggère des pistes pour l'optimisation fiscale.

Faits empiriques. L'analyse des données européennes disponibles sur le patrimoine et les revenus des personnes de plus de 50 ans (Enquête européenne SHARE) montre un certain nombre de faits instructifs. Les ¾ des ménages de plus de 50 ans sont propriétaires de leur logement en Europe. Une fraction non-négligeable de personnes âgées possède un patrimoine immobilier élevé mais des revenus faibles, en comparaison, en raison de l'appréciation des biens immobiliers en zone urbaine, principalement. Par ailleurs, les enfants héritent ou reçoivent de leurs parents assez tard. La moitié des personnes qui ont reçu un transfert d'au moins 5000 euros reçoit un transfert (don entre vifs ou héritage) des parents après l'âge de 50 ans. Si une petite proportion de personnes, non-négligeable, devient propriétaire à l'occasion de l'héritage direct d'un bien immobilier des parents, la moitié des ménages propriétaires environ a acquis son logement avant que le chef de famille ait atteint l'âge de 40 ans. Donc, *les ménages héritent (ou reçoivent de l'aide) en moyenne 10 ans trop tard du point de vue de leur stratégie d'investissement immobilier*.



La figure 1 ci-dessus monte l'histogramme de l'âge au premier transfert reçu (don ou héritage). Source, enquête européenne SHARE.

Le patrimoine immobilier est la forme la plus répandue de détention du patrimoine, puisque le patrimoine financier non-immobilier médian représente 2% du patrimoine médian net. La valeur dont disposent les familles est donc pour l'essentiel immobilisée dans les résidences des parents. Le patrimoine d'un ménage, sous ses diverses formes, tend pourtant à décroître après l'âge de 65 ou 70 ans. D'un autre côté, on observe que les transferts (héritages ou dons entre vifs) jouent un rôle significatif dans l'accession à la propriété en Europe. Les ménages de personnes de plus de 50 ans observés en Europe aujourd'hui, qui ont reçu de leurs parents, sont plus souvent devenus propriétaires lorsqu'ils ont reçu plus jeunes et lorsqu'ils ont reçu beaucoup — ce que le bon sens suggère, mais que la statistique confirme. *L'accession à la propriété n'est pas bien expliquée par le revenu du propriétaire mais la probabilité d'accession augmente significativement avec l'existence et le montant des dons et héritages*. Il est raisonnable de penser que ces personnes, devenues elles-mêmes parents, donneront à leurs enfants des sommes d'argent et des biens qui pourront jouer un rôle déterminant dans la construction de leur patrimoine.

Tableau 1	% de propriétaires*	% des ménages ayant reçu don ou héritage*
Autriche	56	18
Allemagne	53	30
Suède	57	42
Pays Bas	59	28
Espagne	87	17
Italie	78	19
France	72	23
Danemark	61	37
Grèce	84	25
Suisse	52	47
Belgique	79	42
Israël	83	29
Total	**70**	**30**

Source : données Européennes SHARE; *ménages de plus de 50 ans.

Tableau 2	Valeur du logement	Valeur des autres actifs	Taux d'imposition effectif de l'héritage (enfant)		Taux d'imposition effectif de la donation (enfant)	
			Union européenne*	France**	Union européenne*	France**
Cas A	135000 €	33750 euros d'épargne financière	2,8%	7%	3%	7%
Cas B	338000 €	169000 € d'épargne financière	5,75%	15,7%	5%	15,7%
Cas C	540000 €	Epargne 135000 Actif professionnel d'une valeur de 675000 €	5,12%	12,3%	4%	12,3%

*Source : European Commission (2014), *Cross Country Review of Taxes on Wealth and Transfers of Wealth*. Les taux effectifs sont des moyennes sur 27 pays. **France: on étudie le cas d'un enfant unique héritier en ligne directe, l'abattement de 100000 euros s'applique ; le dispositif Dutreil s'applique au transfert de l'entreprise dans le Cas C.

La fiscalité des transmissions par dons entre vifs ou héritages peut bien évidemment jouer un rôle clef dans le timing des dons parentaux et donc dans la distribution des âges d'accession à la propriété. Il y a une diversité de dispositions fiscales en Union Européenne, mais un socle commun de règles est partagé par un groupe central de pays.

L'héritage est taxé dans 18 des 28 pays de l'Union européenne, mais la plupart des petits héritages sont exemptés. La fiscalité des héritages est, en proportion de la recette fiscale totale, la plus élevée en

Belgique (1,37%) et en France (0,97%). Tous les pays de l'Union européenne taxent les donations sauf l'Autriche, le Portugal, la Suède les pays Baltes et la Roumanie. Le Tableau 2 montre des taux d'imposition effectifs moyens qui ont été calculés dans trois cas-types.

Hypothèses, modèle, contrats. L'idée de contrat familial intermédié peut être étudiée dans le cadre d'un modèle, relativement simple, dont on montre la flexibilité et les nombreuses possibilités d'extension. On considère le cas typique d'une mère et de sa fille unique. La mère est veuve et propriétaire de son logement. La fille souhaite accéder à la propriété en empruntant pour acheter son propre logement. Un contrat de crédit classique se fonderait seulement sur les revenus de la fille et prendrait le logement de la fille comme garantie. Cette dernière héritera un jour de la maison de sa mère si la mère ne lui fait pas un don de son vivant.

Nous faisons ici trois hypothèses importantes. Dans une population de veuves propriétaires avec une fille unique, les individus se caractérisent d'abord par un certain degré d'*aversion pour le risque* (mère et fille sont prêtes à payer une prime d'assurance pour réduire la variabilité de leur revenu et de leur consommation de divers biens, dont le logement). Ensuite, la mère se caractérise par un certain degré d'altruisme, c'est à dire un certain souci du bien-être de sa fille (elle est concernée, à un certain degré, par la taille du logement de sa fille et plus généralement par son niveau de vie). Il peut y avoir des mères plutôt égoïstes et des mères plutôt généreuses, l'hypothèse est qu'il y a un certain degré, non nul, d'altruisme parental. Il peut aussi exister des filles qui voudraient aider leur mère. Pour cette raison, les choix des mères s'apparentent à (et sont traités comme) des décisions économiques collectives qui peuvent comporter un élément de redistribution (une mère riche augmentera sa satisfaction en donnant à une fille relativement plus pauvre). Enfin, la mère est supposée très attachée à son logement, elle ne veut pas le quitter, et elle ne veut pas le quitter pour un logement plus petit ni même, dans une certaine mesure, pour un logement plus grand mais différent. Cette hypothèse est assez réaliste et semble caractériser (à des degrés divers) un grand nombre de personnes âgées.

Le contrat optimal est offert par un banquier-assureur qui pratique un calcul financier (escompte) et un calcul d'actuaire, fondé sur les probabilités de survie de la mère (et l'évaluation rationnelle d'autres risques). Cet intermédiaire diversifie ses risques en gérant un portefeuille de contrats familiaux suffisamment grand. Le modèle que nous étudions met l'accent sur le problème d'assurance central qui est ici le risque de longévité de la mère. Il y a par ailleurs des risques sur le prix futur des biens immobiliers, des risques de chute dans la dépendance dus au grand âge, et des risques de perte (ou de fluctuation) des revenus que nous laissons de côté pour les besoins de la clarté de l'exposé. Nous établissons que le contrat optimal, sous nos hypothèses, consiste à partager le revenu et la richesse totale de la fille et de la mère, au cours du temps, de la manière qui suit. La mère vend sa maison en viager à la banque; elle reste chez elle comme usufruitière ; elle touche une pension de la banque jusqu'à sa mort. La banque fait à la fille un crédit immobilier classique qui comporte une subvention ou un rabais équivalent à un certain don de la mère, qui peut être échelonné au cours du temps, et sur toute la durée du crédit. La mère a pu donner une partie du bouquet de son viager à sa fille, en franchise de droits de mutation à titre gratuit, si la partie du bouquet transférée est inférieure aux seuils d'exemption en vigueur. Enfin, la fille est totalement immunisée contre les conséquences d'un décès (ou de la maladie, suivie du décès) de sa mère : son revenu net de ses remboursements à la banque ne changent pas après le décès de sa mère ; en substance, la fille a déjà touché son héritage.

La mère et la fille auraient-elles pu mettre en œuvre cet arrangement et « régler leurs affaires » par leurs propres moyens, en combinant des outils classiques ? Pas tout à fait, ni si facilement. La mère aurait pu vendre son logement en viager de gré à gré, avec les risques et difficultés que cela comporte. L'entité banque-assurance, gérant un portefeuille immobilier, serait beaucoup plus efficace dans ce domaine. Elle peut ensuite transférer tout ou partie de son bouquet ou de sa rente viagère, en évitant les impôts sur les dons entre vifs, au besoin en donnant du liquide chaque mois à sa fille pour l'aider à payer son crédit immobilier. La fille aurait pu prendre un crédit immobilier seule, mais pas aussi facilement, ni dans des conditions aussi bonnes, car la richesse de sa mère ne serait pas prise en compte. Enfin et surtout, l'aide de la mère cesserait avec sa mort, ce qui n'est pas le cas avec le contrat familial intermédié, car le banquier qui est aussi assureur sur la vie, peut aider la fille après la mort de la mère. C'est un avantage-clef de l'intermédiation.

Simulations. Nous calculons le contrat optimal pour un échantillon artificiel de familles, calibré pour être représentatif d'une classe moyenne aisée de propriétaires-occupants vivant en région parisienne, à titre d'illustration (micro-simulation). On calcule pour chaque famille le profit maximum (en termes actuariels, espérés et actualisés) qui peut être réalisé et partagé entre les parties à l'occasion de la mise en place du contrat familial optimal. On montre que la profitabilité de ces contrats est positive et non-négligeable ; que la fille achète un logement en moyenne plus grand qu'en l'absence de contrat familial ; que la satisfaction (utilité ou bien-être) de la famille augmente. Pour calculer le profit maximum, on compare l'utilité de la mère sous contrat familial avec son utilité si elle ne peut disposer que d'outils classiques (option externe de la famille). Cela permet de calculer le surplus actualisé maximum qui peut être extrait d'une famille et d'étudier sa distribution dans la population.

Passage à la pratique. On voit bien que ce type d'arrangement est potentiellement plus intéressant pour les familles s'il comporte une assurance contre le risque de dépendance, permettant, par exemple, de lisser les frais d'une maison de retraite de bon confort, où ceux d'une assistance à domicile. De même, il peut aussi être employé pour résoudre, avant la mort des parents, un problème de partage entre frères, sœurs et parent survivant. Enfin, le contrat peut « sauter » une génération et servir à aider les petits enfants. Ce type de contrat peut aussi être adapté pour aider un enfant ou petit enfant à monter un commerce ou une entreprise, et pour financer les études d'un petit enfant. Ces derniers cas méritent une réflexion spécifique. L'articulation de l'arrangement familial avec l'assurance-vie telle qu'elle se pratique en France mérite également une réflexion spécifique. L'intuition que nous soumettons à la réflexion du lecteur, c'est que les obstacles comportementaux, les réticences à la souscription de contrats viagers (énigme de la sous-annuitisation), le problème du recours limité au viager ou aux prêts viagers hypothécaires de type anglo-saxon, peuvent être plus aisément levés ou atténués dans le cadre de la définition d'un contrat familial intermédié. La façon de présenter ce type de contrat et ses avantages à la clientèle (framing) mérite une étude particulière.

Intermediated Family Contracts*

Robert J. Gary-Bobo, Meryam Zaiem

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Abstract

We study intermediated family contracts. These contracts are used to share the family's wealth and income among its members and to transmit wealth from one generation to the next, with the help of a banker-insurer, according to a multi-period plan. Family arrangements can be defined as a bundle of bilateral contracts signed simultaneously with a bank by several members of a family. These contracts may be contingent on a number of publicly observable events, like a person's death; they can also be analyzed as a bundle of financial products, combining equity release and home loans (reverse and standard mortgages), annuitization (longevity insurance), consumption smoothing and long-term care insurance. A secondary benefit of such a product may be tax optimization. The properties of optimal intermediated family arrangements are studied under three main assumptions: parents and children are risk-averse; parents are altruist to a certain degree; the elderly are owner-occupiers and strongly attached to their homes. We used calibrated simulations to study the welfare and comparative statics properties of the optimal contract in a population of families. Simulations provide bounds on the surplus that can be extracted by the intermediary, by means of a family contract, *i.e.*, on the profitability of a given family for the banker-insurer.

Keywords : Bequests; Aging; Annuities; Inter-vivos transfers; Reverse Mortgages; Taxation; Housing equity.

*CREST, ENSAE, 15 boulevard Gabriel Péri, 92245 Malakoff cedex, France; E-mail:robert.gary-bobo@ensae.fr

1 Introduction

In the present article, we study intermediated family agreements. These contracts are used to share the family's wealth and income and to transmit wealth from one generation to the next, with the help of a banker and (or) an insurer. The family arrangements that we study can be defined as a bundle of contracts signed simultaneously with a bank by several members of a family. In legal terms, it may not be necessary to sign a multilateral agreement, but the banker may require that the family members involved sign a two-sided contract with the bank at the same moment, and the contracts may be contingent on a number of publicly observable events, like a person's death. The family contracts can also be analyzed as a bundle of financial products, combining equity release and home loans (reverse and standard mortgages); annuitization (longevity insurance), consumption smoothing and long-term care insurance. Intermediated family contracts solve several important problems at the same time. The most important of these problems is the specification of an intertemporal plan to share the parents' estate with the children. This is why these contracts could also be called *notarial*. A secondary benefit of such a product may be to save on some taxes.

We study these intermediated notarial contracts under several key assumptions. First, the older member of the family (hereafter called the *mother*), is supposed to be an altruist. The mother wants to maximize a weighted sum of her utility and the utilities of her children. Second, a number of risks, and in particular, the mother's longevity risk and the mother's risk of a severe health accident requiring long-term care, as well as market risks like house-price variability, can be diversified and mutualized by a banker-insurer, who holds large bundles of family contracts. Third, we assume that the mother owns her residence and that she is strongly attached to her house: the disutility of moving or the disutility of house downsizing are supposed to be very high. The banker can therefore also offer a liquidity service, unlocking the mother's housing equity by means of a reverse mortgage, a home reversion plan or a French *viager* contract, while permitting the mother to stay in her home. The money released can be used to schedule gifts to the next generation, and the children can at the same time borrow to buy their own homes.

We characterize the set of optimal intermediated family contracts in a simple model with a finite horizon, discrete time, risk-averse and altruist agents. The optimal solution may depend on the tax system, but if the tax rate on bequests is positive, the mother's home equity should be entirely liquidated by means of a *viager* contract (or home reversion plan); direct inter-vivos gifts to the children should be zero but the children's mortgage payments can be subsidized; the mother's assets can simultaneously be annuitized and the children are fully hedged against the mother's longevity risk. The model can be generalized to study various related questions. In particular, the analysis can be extended to study the case of multiple children or heirs and multiple risks.

We then calibrate the model to explore the properties of family contracts by means of numerical simulations. We discuss the welfare gains of these contracts, and show how the surplus can be shared and extracted by financial institutions. The simulations are calibrated using French data on income and wealth, and we focus on seniors living in the Paris region as an illustrative example. Our simulations provide an evaluation of the profitability of improvements in the efficiency of risk-sharing due to the intervention of financial intermediaries in family arrangements.

Finally, we discuss the practical problems posed by the design of intermediate family contracts, focusing on the potential behavioral obstacles to the marketing of such products. The contribution of the paper is not to emphasize the usefulness of equity release instruments, that has been studied by various authors (see below), but to explore the consequences of optimal risk-sharing between family members in terms of product design (and the associated financial engineering).

The present paper starts with an empirical section in which we use the European SHARE survey on the elderly in Europe to show a number of statistical facts related to the transmission of wealth within families.

Literature

The life-cycle theories of consumption and savings have generated a number of puzzles. In particular, the observed behavior of the elderly seems to contradict standard theory, challenging the individual rationality assumptions. One of the well-known puzzles is the absence of asset decumulation, or dissaving, among retired and aging households. It seems that the elderly do not wish to downsize their housing consumption and do not even seem to be willing to borrow against home equity to finance consumption. These facts have been discussed a long time ago. Except for Social Security and some pension-related assets, housing equity is the most important asset of a large fraction of older citizens in Europe and in the United States. Venti and Wise (1989), (1990) showed that the elderly are as likely to move to a larger house as into a smaller one. This seems to contradict the prediction of life-cycle theory according to which old agents should reduce savings and housing consumption. In general, retired households resist financial downsizing in a way that seems "irrational" (see Dynan *et al.* (2004), De Nardi *et al.* (2010)). Further study of this question shows that, in the absence of changes in household structure (for instance, the death of a spouse), or other shocks, like the need to enter a nursing home, most elderly families are unlikely to move (see Venti and Wise (2004)). It is reasonable to assume that retired individuals are strongly attached to their homes. This deep-rooted attachment has been studied by social scientists (see for instance Curry *et al.* (2001)). Psychological factors certainly play a rôle, as introspection may reveal, but home-ownership is also objectively providing a form of insurance against various risks. Firstly, ownership is a hedge against upward variations in future house prices; secondly, liquidating the house and entering the rental market exposes the household to rent variability, and this latter risk cannot be insured. Sinai and Souleles (2005) have shown that home-ownership rates are higher in areas with greater variability of rental prices. In his comments on Venti and Wise (2004), Jonathan Skinner (2004) recalls that the most common reason for moving is poor health, but that the second most common reason is to move closer to family (see Choi (1996)). This may explain why moving does not necessarily cause downsizing. Skinner (1996) also discusses the role of (psychological and real) moving costs.

It seems relatively well-established that the commitment of the elderly to their homes is also due to the fact that these homes are a store of value, and more precisely, a substitute for *long-term care insurance*. There exists a literature on the reasons for which long-term care insurance is underdeveloped (see, *e.g.*, Brown and Finkelstein (2007)). Medicaid and, more generally, public insurance systems may be responsible for some crowding out of the demand, but the main reason seems to be the fact that home-ownership plays the role of long-term care insurance for many people (see Thomas Davidoff (2010)).

Venti and Wise (2004) put the emphasis on another important fact: the apparently limited recourse to housing equity-release instruments, and mainly to reverse-mortgage contracts that are available in the United States since the mid 90s. Many older households have substantial wealth locked in illiquid housing. They should normally like to release this wealth to finance consumption. Reverse mortgages in the US, the UK and other countries (to a certain extent, *viager* contracts in France) attract a limited number of clients only, in spite of important potential benefits — at least in theory. In many countries, there is a significant segment of “income-poor” and “house-rich” individuals. For international comparisons of housing equity adjustment decisions, see Chiuri and Jappelli (2010), Fornero and Rossi (2012). As mentioned by Skinner (2004), Davidoff (2010) and others, the limited recourse to reverse mortgage loans may not be irrational, but, on the contrary, may be due to several clearly understandable factors: the insurance properties of home-ownership on the one hand, and tax optimization on the other hand, since real property is a “safe haven” under some tax systems.

This does not mean that behavioral economics considerations do not contribute to the explanation of some facts: financial literacy considerations (and risk aversion) have been used to explain the limited recourse to reverse mortgages (again see Fornero and Rossi (2012); on mortgages in general, see Cox *et al.* (2015)). Because of imperfect insurance against expenditure shocks due to health accidents, the elderly may prefer a well-designed line-of-credit plan to a housing-equity release contract (Fratantoni (1999)). The stylized fact that equity release loans are characterized by low take-up rates may have to be revised because the reverse-mortgage market witnessed substantial growth in the mid 2000s, particularly in the United States (see Hui Shan (2011), Sinai and Souleles (2008)). Higher house prices, generating windfall gains for many homeowners, may finally persuade large fractions of the potential borrowers to engage into equity release (on the rise of house prices, see Gleaser *et al.* (2005)). Recent research will lead to improvements in financial engineering techniques and, among other things, to a better understanding of the borrowing limits of reverse mortgage contracts (see, *e.g.*, Pu, Fan and Deng (2014)). In some countries, such as France, the reverse mortgage market is totally underdeveloped. The French *prêt viager hypothécaire* has been legalized by an act passed in 2006. This kind of loan is strictly regulated and French banks cannot advertise it. In contrast, the *viager* contract is not limited.

The degree of financial development of a country may be highly correlated with the degree of financial literacy of its inhabitants. Economic agents typically make mistakes and do not behave as predicted by normative theory, but when explanations are offered to them, in general, they understand the benefits of new products. Even if they don’t understand the contracts well, at some point, they simply start to mimic successful neighbors who did choose these products before them. But either kind of learning takes time. For that reason, the development of reverse mortgages may become impressive in a few years from now, even in financially under-developed countries, simply because they are useful and enhance welfare, as predicted by normative theory, and since people will eventually understand their usefulness. Note that limits to the development of equity-release contracts can also be due to the presence of bequest motives, that we discuss below.

The French market for housing-equity release is still in its infancy. In this country though, the *viager* contract is a traditional mode for the sale of a house. Yet, this market is residual because individual buyers have to bear the longevity risk of the seller, which is a typically sub-optimal way of sharing risks. These risks should obviously be borne and diversified by insurance companies and banks. In France, financial intermediaries started to buy houses in exchange for a *viager* contract in the recent months. An experimental Real Estate Investment Trust (*i.e.*, *Certivia*) has been launched in 2014, under the aegis of the Caisse des Dépôts, with limited amounts of capital; the private financial sector has just started to intervene as an intermediary in the *viager* market, with a few newly created funds, that is, more precisely, with FPCIs (in French, *Fonds Professionnel de Capital Investissement*) like *Silver Estate*. A few economists advocated the development of intermediated *viager* markets in France (see Masson (2014)). The social benefits of this type of financial innovation could indeed be huge, and to the best of our knowledge, they have not been estimated by means of rigorous methods. The economic literature on the French traditional *viager* market is recent and, to the best of our knowledge, papers on this topic are rarities (see Février *et al.* (2012)).

The *annuity puzzle* is another important behavioral anomaly (see, *e.g.*, the survey of Benartzi *et al.* (2011)). Older households do not *annuitize* their wealth, while theory predicts that they should. The classic result is due to Yaari (1965): a risk-averse agent without a bequest motive should completely annuitize her wealth, that is, sell her assets in exchange for a sequence of certain payments that will be terminated after the end of the agent's life. The use of annuities is rational because they protect old agents against the risk of outliving their income. In the economics of life insurance, those who live longer than the average receive a subsidy from those who die early. Davidoff *et al.* (2005) present conditions for the optimality of full annuitization under market completeness which are more general than Yaari's assumptions. In incomplete markets, they show that consumers will generally want to annuitize a significant fraction of their wealth. There is also a debate on the reasons for which these markets are underdeveloped, and this is still an open research question (again, see Benartzi *et al.* (2011)). Recent research has focused on behavioral obstacles to annuitization. The behavioral theories of Kahneman and Tversky (Khaneman (2011), Kahneman and Tversky (1979)) and Richard Thaler (see Thaler (1999)) have been applied to understand the annuity markets (see, *e.g.*, Hu and Scott (2007)). Scott, Watson and Hu (2011) study annuity design problems and suggest that some innovations may increase participation in the annuity market and allow for significant welfare improvements. Again, bequest motives and the desire to keep real estate assets as a hedge against large shocks, as well as the rigid or irreversible consequences imposed by a limited set of available options may explain incomplete annuitization and limited participation.

On these themes, see Campbell (2006), who argues that if some observed facts are unquestionably behavioral anomalies in a certain sense, their origin is often unclear. It may be that theory, and the economic models, are too simple to capture the complexity of individual decision problems; the data sources are too limited to convincingly identify some effects; and of course, some —but not necessarily a majority of— individuals lack knowledge and simply make mistakes. It follows that the data may lead to rejection of a textbook model but that a more sophisticated theory relying on rationality assumptions would fit the data reasonably well. Some key parameters in life-cycle and financial theories are typically hard to identify in the econometric sense. The environment of households is constantly changing, in particular because of innovations and due to the development of financial markets, and individuals may be learning slowly, or many of them just imitate the behavior of others, so that adjustment to economic equilibrium in the standard sense takes time. We keep these difficulties in mind in what follows.

The next important point in the following analysis is the existence of parental altruism and bequest motives. Economists hesitate about the observed bequests: are they random, purely unintentional transfers or do they reflect the existence of a bequest motive? It is likely that observed bequests are a mixture of intentional and unintentional transfers of wealth. Without bequest motives, the life-cycle theories predict that bequest will be nonzero, because death is not perfectly predictable. A further question is the importance of individual mistakes, behavioral biases and inaccurate planning in the distribution of inheritance. Economic theory then hesitates about the appropriate model for the bequest motive. In a nutshell, is it better to assume that individuals' preferences are characterized by "joy-of-giving", that is, an *ad hoc* utility for the gifts made to children, or should we model individuals as *pure altruists*? In this context, pure altruism means that utility functions depend on the utility of children, leading to dynastic and recursive formulations of utility, since the utility of the father depends on the utility of the son which in turn is a function of the grandson's utility, etc.

The importance of the "joy of giving", altruism and bequest motives has been recognized a long time ago by economists (see, e.g., Becker (1974), Andreoni (1989), Abel and Warshawsky (1988), Laferrère and Wolff (2006)). The distinction between the various sources of inheritance flows is of course crucial for the theory of taxation (see, *e.g.*, among recent papers, Cremer and Pestieau (2006), Diamond (2006)), and this may explain the failure of Ricardian equivalence. If bequests are mainly accidents, they can be taxed with no incidence on savings.

The evidence concerning the existence of a bequest motive supports the hypothesis (see, *e.g.*, Kopczuk and Lupton (2007)). There also exists an important econometric literature on family transfers and the pure altruism hypothesis. The latter assumption is rejected by some tests (Altonji *et al.* (1997)), but currently, there is a lack of consensus on this question (see Kaplow (2001), Kopczuk (2013)). Again, a possible explanation for the lack of asset run-down and under-annuitization may be aversion to the prospect of having insufficient wealth to pay for private long-term care, and therefore, of needing public care. Ameriks *et al.* (2011) formulated the hypothesis that there exists a *public-care aversion* among the elderly in the United States. These authors endeavored to disentangle the effects of *public care aversion* and bequest motives on the degree of wealth decumulation during retirement. They found that *public-care aversion* is a significant driver of precautionary savings. These fears may have equivalents in other countries. According to Ameriks *et al.* (2011), there is a "powerful interaction between interest in annuities and the institutions that provide for long-term care. (...) The demand for annuities would be far higher if they included some acceptable form of long-term care insurance". They also find that bequest motives are "more prevalent and spread deeper into the middle class than is generally believed", and they find evidence for heterogeneity in these motives.

In the following, Section 2 provides some descriptive statistics and econometric tests, based on European data, namely, the SHARE survey. Section 3 presents the institutional setup: the existing housing-equity release instruments and inheritance and property taxation rules (in Europe). Section 4 presents a simplified version of our family contract model. A number of extensions and generalizations are discussed in Section 5. Section 6 presents various numerical simulations of the model, based on its multi-period extension. As a conclusion, Section 7 discusses contract design and the role of behavioral obstacles.

2 Descriptive Statistics and Econometric Tests

We begin with a presentation and discussion of some descriptive statistics and econometrics based on SHARE data. SHARE is a large scale survey of the health and wealth of persons aged 50 and more in Europe. In the following, we particularly use the first wave of SHARE, with more than 30000 individuals surveyed between 2004 and 2005. We will focus on a number of features of the survey that are directly relevant to our later analysis: income (pensions), wealth, real property, intergenerational transfers (bequests and inter vivos gifts), accumulation and decumulation of wealth.

We work with a sample of 20505 individuals, called *principal respondents*. Each principal respondent is attached to a household. This is due to the double structure of the SHARE survey: the data provide information on individuals, but also on the household to which they belong (generally on the spouse). Some variables, (*e.g.*, age, gender, health status) refer to the individual called *principal respondent*. But other variables, like, wealth, financial assets, real estate assets, debt, income are computed for the household attached to the principal respondent. Among these individuals, we more particularly studied those who received money from their parents. On the SHARE survey and methodology, see Börsch-Supan et al.(2005a), Börsch-Supan and Jürges (2005b).

Table 1 breaks the 20,505 households by country of residence. In this sample, among principal respondents, there are 10,732 women (slightly more than 50% of the sample). Table 2 gives the marital status of the surveyed individuals. The number of widows is of course particularly high, since the principal respondent's age is distributed between 50 and 103. The size of the household is distributed as indicated in Table 3. Table 4 gives the distribution of the health status in 5

Table 1: Country of Residence

	Count	%
Austria	1,177	5.7
Germany	1,942	9.5
Sweden	2,111	10.3
Netherlands	1,908	9.3
Spain	1,646	8.0
Italy	1,732	8.4
France	2,033	9.9
Denmark	1,163	5.7
Greece	1,972	9.6
Switzerland	685	3.3
Belgium	2,498	12.2
Israel	1,638	8.0

Source: SHARE data set, first wave, principal respondents only.

categories (from very good to very bad).

We now study the housing variables. Each principal respondent has a principal residence. Table 5 gives the distribution of the housing status. In the sample, 70% of the households are owners of their principal residence. Table 6 explains how the residence was acquired. We see that the house was received as a bequest or a gift in 10% of the cases.

To obtain a better understanding of access to the status of owner-occupier, a glance at the distribution of the age at which the residence was acquired is useful. Fig. 1 gives the histogram of this distribution, based on SHARE data. Strictly speaking, Fig. 1 gives the probability of acquisition at an age denoted t_h, given that the age a at the moment of the survey is greater than 50 and given $t_h \leq a$. It follows that the reported frequencies are biased estimates of the probabilities of acquiring a home at ages $t_h > 50$, because data are censored for individuals of age $a > 50$ that would buy their homes only in the future, at an age $t_h > a$. The associated survival distribution of the acquisition age could be estimated by the Kaplan-Meier method, but the estimates of probabilities would be imprecise for ages greater than 50. The reader must remember that probabilities of acquisition after 50 are somewhat under-estimated by the histogram frequencies. We see that roughly 60% of the studied sample bought their house between 30 and 50, while 20% became owners before 30.

We now look at transfers and try to see if the data suggests that transfers played a role in helping the households building up their wealth. The data doesn't indicate if the money received, or the assets received by households take the form of a bequest or of an *inter-vivos* gift. This is why we describe gifts and bequests simply as *transfers* below. These transfers are concentrated. Most of the surveyed individuals never received a transfer greater than 5000 euros.

Table 2: Marital Status

	Count	%
married and living together with spouse	12,501	61.0
registered partnership	302	1.5
married, living separated from spouse	298	1.5
never married	1,409	6.9
divorced	1,749	8.5
widowed	4,233	20.7

Source: SHARE data set, first wave, principal respondents only.

Table 3: Size of the Household

	Count	%
1	6,018	29.3
2	9,785	47.7
3 or more	4,702	22.9

Source: SHARE data set, first wave, principal respondents only.

Table 4: Health Status

	Count	%
very good	2,924	14.3
good	6,071	29.6
fair	6,944	33.9
bad	3,404	16.6
very bad	1,146	5.6

Source: SHARE data set, first wave, principal respondents only.

Table 5: Ownership and Tenancy Status

	Count	%
owner	14,307	69.8
member of a cooperative	463	2.3
tenant	4,563	22.3
subtenant	74	0.4
rent free	1,071	5.2
usufructuary (France only)	14	0.1

Source: SHARE data set, first wave, principal respondents only.

Table 6: How the residence was acquired

	Count	%
purchased or built solely with own means	11,612	78.8
purchased or built with help from family	1,009	6.8
received as a bequest	1,446	9.8
received as a gift	201	1.4
acquired through other means	463	3.1

Source: SHARE data set, first wave, principal respondents only.

Figure 1: Histogram of age at which house was acquired

Table 7: Share of households who received a transfer

	% owners	% households who received a gift or bequest
Austria	56	18
Germany	53	30
Sweden	57	42
Netherlands	59	28
Spain	87	17
Italy	78	19
France	72	23
Denmark	61	37
Greece	84	25
Switzerland	52	47
Belgium	79	42
Israel	83	29
Total	70	30

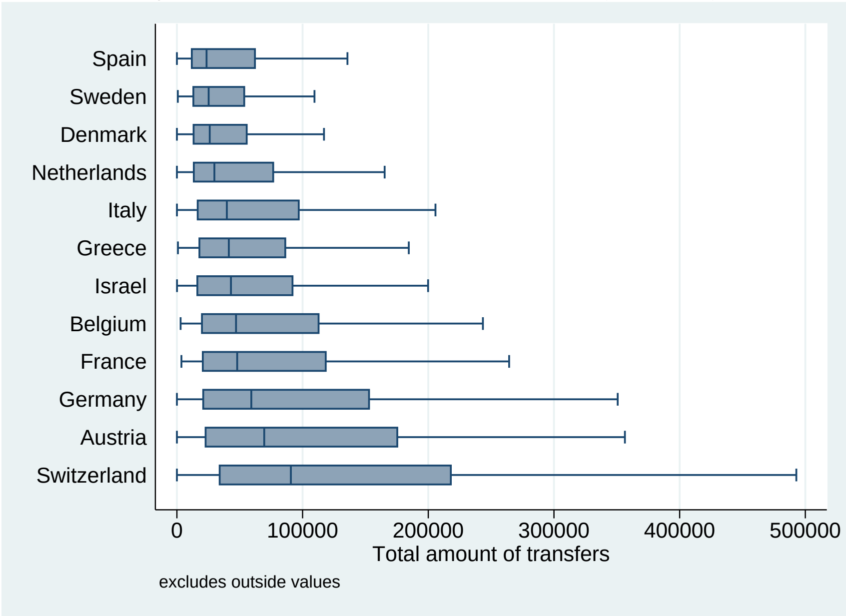
Source: SHARE data set, first wave.

We found 6274 individuals who received a transfer greater than 5000 euros at least once in their life. There are various missing data problems: for instance, we sometimes do not know when the transfer was received. To study the timing of the transfer (when it was received), and the amount, we selected the largest sub-sample without missing observations of the relevant variables.

Table 7 shows the proportion of owners and the percentage of those who received at least one transfer, broken down by country. The European average shows that 70% of the SHARE households are owner-occupiers and that 30% received a transfer. Fig. 2 breaks the population by country of origin and shows the boxplots of the distribution of gifts, countries being ranked by the median transfer, in ascending order. There are many outliers the values of which are not depicted. The figure shows striking differences between some countries — for instance, between Sweden and Switzerland. Fig. 2 also shows that dispersion (inter-quartile range) varies widely from one country to the next. It seems that richer countries and(or) countries with less redistribution by taxation have larger medians and a much larger dispersion of their distribution of transfers — with inheritance-tax havens at the bottom of the list.

The survey records a list of at most five transfers per respondent with the time at which they were received. We now look at the distribution of the age at which the principal respondent received the first and the second transfers (if they exist). Transfers are ranked by magnitude, but the data can be rearranged to provide a time sequence. Figure 3 depicts the distribution of the age at which the first transfer (not necessarily the greatest) was received. In this figure and the next, the same *caveat* is needed: the displayed frequencies are biased estimates of the corresponding probabilities, because data is censored, for persons with ages strictly greater than

Figure 2: Boxplot of total transfers, by country



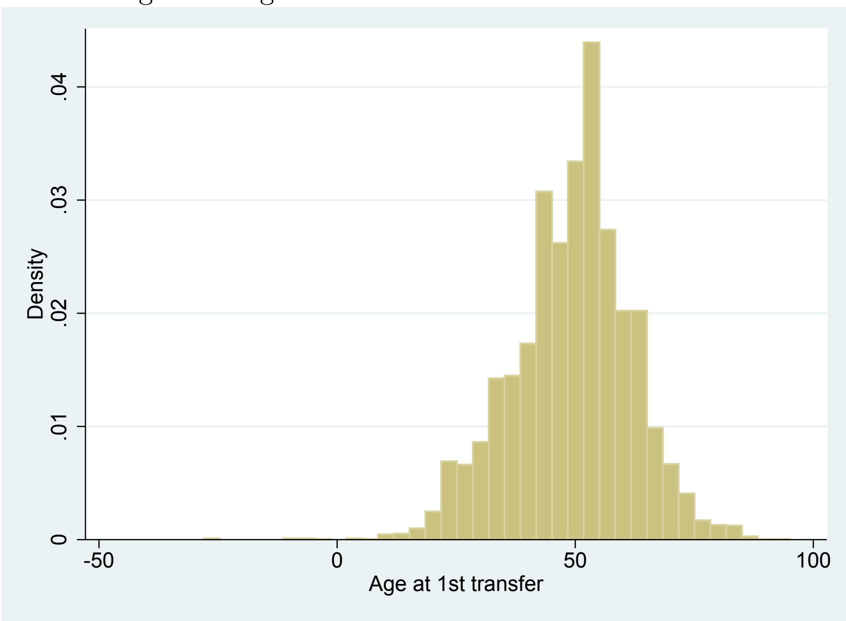
Source: SHARE data set, first wave.

50 at the time of the survey may receive a transfer in the years following the survey (but Kaplan-Meier survival estimates would not be very precise for old ages).

On Fig. 3 we see that people receive help from their parents at 50 on average. The median age is around 50 (probably slightly overestimated). In addition, 75% of the individuals receive their first transfer after 42 (at least). We can safely conclude that help coming from the parents or bequests arrive late in life. Presumably, young adults would have needed help during their thirties. Reality shows that they have to wait until they themselves have raised their children and bought a house before receiving a transfer that is very likely to be a bequest. It is not easy to know which of the transfers reported in the data are in fact bequests, when the data is missing. When the individual's parents are still alive, we know that the transfer was in fact a gift. Figure 4 gives the observed frequency distribution of the age at which the second transfer was received (second in historical order). The median age of the second transfer is slightly larger, at 54. Its standard deviation is smaller than that of the distribution reported on Figure 3 (10 years instead of 12.5), and 75% of the individuals receive their second transfer after 48 (at least). Again, the distribution gives a slightly biased view of the corresponding probabilities because of censoring. In the sample, we also observe that 81% of the individuals receive only one transfer above 5000 euros and 97.5% receive only two such transfers. This is why we focused on the first two transfers. Presumably, the vast majority of all recorded transfers are bequests.

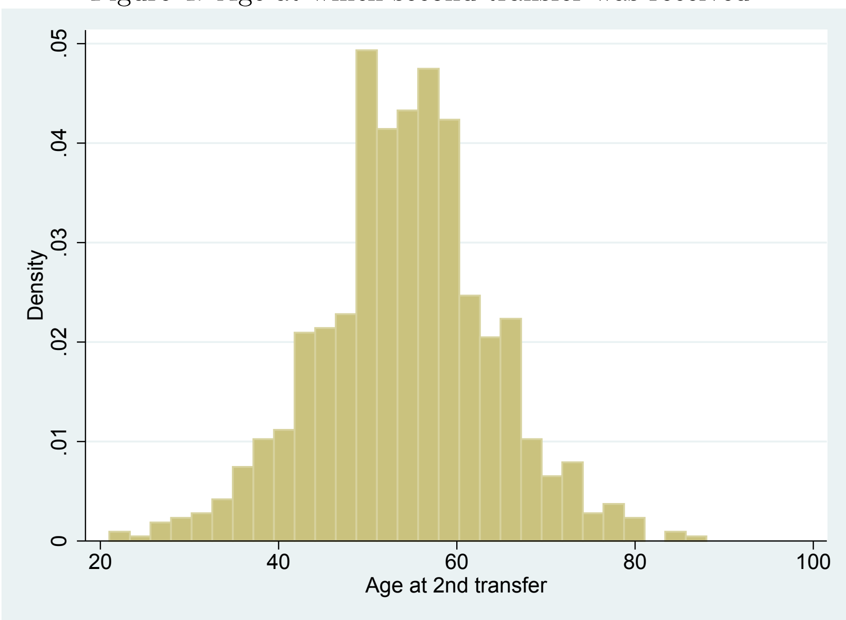
We now study the age at which the main property was acquired as possibly determined or caused by the transfers. Figure 1 depicts a frequency distribution where 99% of the mass is above 19 years old, the mean is 41 and the median is 39. The typical individual buys a home (at least) 10 years before receiving a bequest from his(her) parents (with the caveat due to censoring).

Figure 3: Age at which first transfer was received



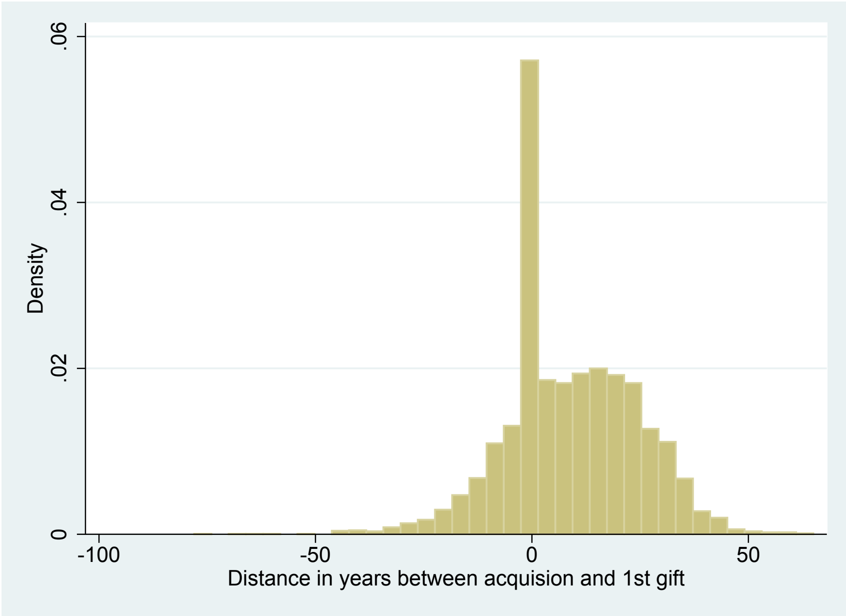
Source: SHARE data set, first wave.

Figure 4: Age at which second transfer was received



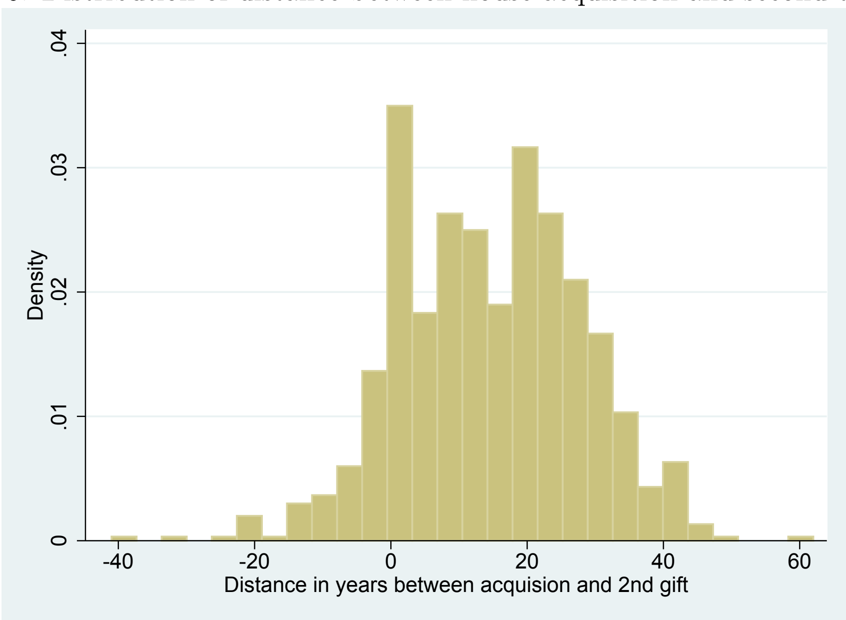
Source: SHARE data set, first wave.

Figure 5: Distribution of distance between house acquisition and first transfer



Source: SHARE data set, first wave. Distribution of distance, in years, between the year of house acquisition and the year the first transfer was received

Figure 6: Distribution of distance between house acquisition and second transfer



Source: SHARE data set, first wave. Distribution of distance, in years, between the year of house acquisition and the year the second transfer was received

Table 8: Summary statistics about wealth

	Mean	Median	Standard deviation
Total value transfers	87,100	37,089	149,815
Financial assets	55,189	12,000	108,350
Total assets	317,932	200,000	451,700
Debt	26,476	0	56,423
Net wealth	282,679	174,100	435,549

Source: SHARE data set, first wave. Figures in 2004 euros.

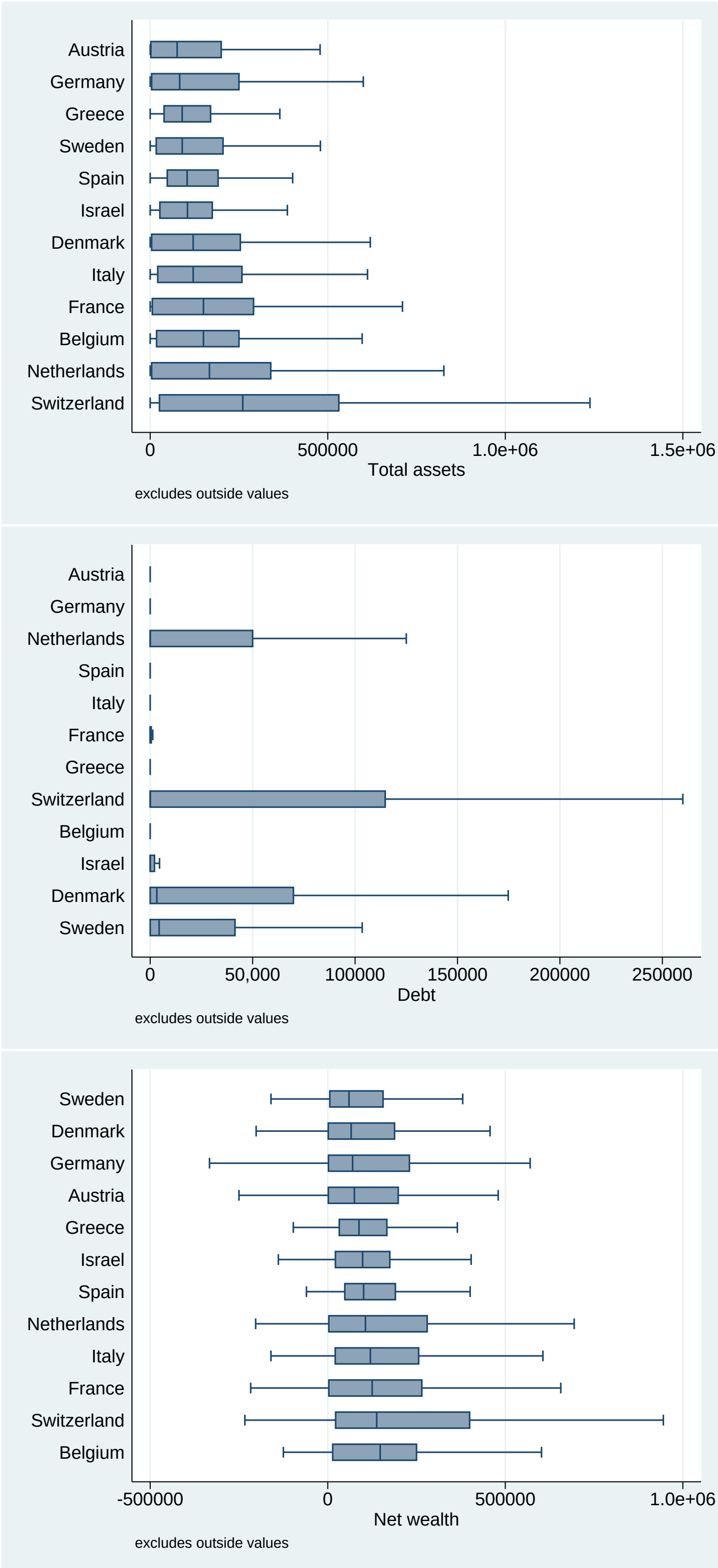
We now consider the possible relationship between inter-generational transfers and property acquisition. Figure 5 depicts a remarkable distribution, namely that of the difference (in years) between the main residence's acquisition date and the moment of the first transfer. The distribution displayed on Fig. 5 is obviously the mixture of a mass at point 0 (hereafter called the "peak") and a regular bell-shaped density. The distribution's mean is 9 years, with a standard deviation of 15 years, and 50% of the individuals wait at least 7 years, after the transfer, before the acquisition of their property. Figure 6 shows the equivalent distribution for the second transfer, and there is no peak. The latter distribution is less "regular".

It is now interesting to study what characterizes individuals in the peak of Fig. 5. Out-of-peak and within-peak individuals are strikingly different. In essence, 83% of the off-peak individuals purchased or built their houses "solely with their own means", while 50% of the within-peak individuals in fact obtained their property directly as a bequest. Only 7% of the individuals declare that they purchased their home with the help of their family.

We will come back below to the study of transfers but we first describe the distribution of income and wealth in the sample. Table 8 gives the medians and means of several important variables: the total value of transfers; the value of a household's financial assets; the household's total assets (including real estate); the household's debt and the household's net worth. It is immediate that debt is almost negligible, as a first approximation, for the vast majority of the households. The median value of debt is zero. Median financial assets are also small in proportion to total assets: housing is the only important asset for most people. The median transfer is not negligible and it may cause (or simply be correlated) with the acquisition of assets (real estate). The dispersion of financial assets and total assets is huge, reflecting a number of well-known facts about inequality.

We will learn more about inequality among seniors in Europe by looking at boxplots of the key variables related to wealth, by country. These statistics are given by Fig. 7, in which the medians have been ranked in ascending order. Table 9 gives the corresponding figures and permits one to compare the medians and the means of the key financial variables. Fig. 8 gives another view of the key variables, averaged over all survey countries but broken by size of the household.

Figure 7: Total assets, debt and net worth, by country



Source: SHARE data set, first wave. Figures in 2004 euros.

Table 9: Means and medians of total assets, debt and net worth

	Total assets		Debt		Net worth	
	Mean	Median	Mean	Median	Mean	Median
Austria	144,930	75,900	3,996	0	139,932	75,000
Germany	181,040	83,000	10,393	0	165,403	70,000
Sweden	181,002	90,160	32,047	4,357	142,407	59,911
Netherlands	253,286	166,800	33,178	0	214,346	106,175
Spain	214,820	104,000	4,606	0	207,027	101,000
Italy	226,387	121,000	2,963	0	220,529	120,000
France	254,343	150,000	9,593	0	239,917	125,000
Denmark	239,515	120,987	43,437	3,226	177,010	65,871
Greece	146,872	90,000	2,726	0	143,464	88,000
Switzerland	424,383	260,722	57,800	0	335,958	137,857
Belgium	220,073	150,000	4,091	0	210,175	147,600
Israel	179,120	104,895	7,495	0	170,322	98,252
Total	212,447	110,288	14,806	0	190,630	96,790

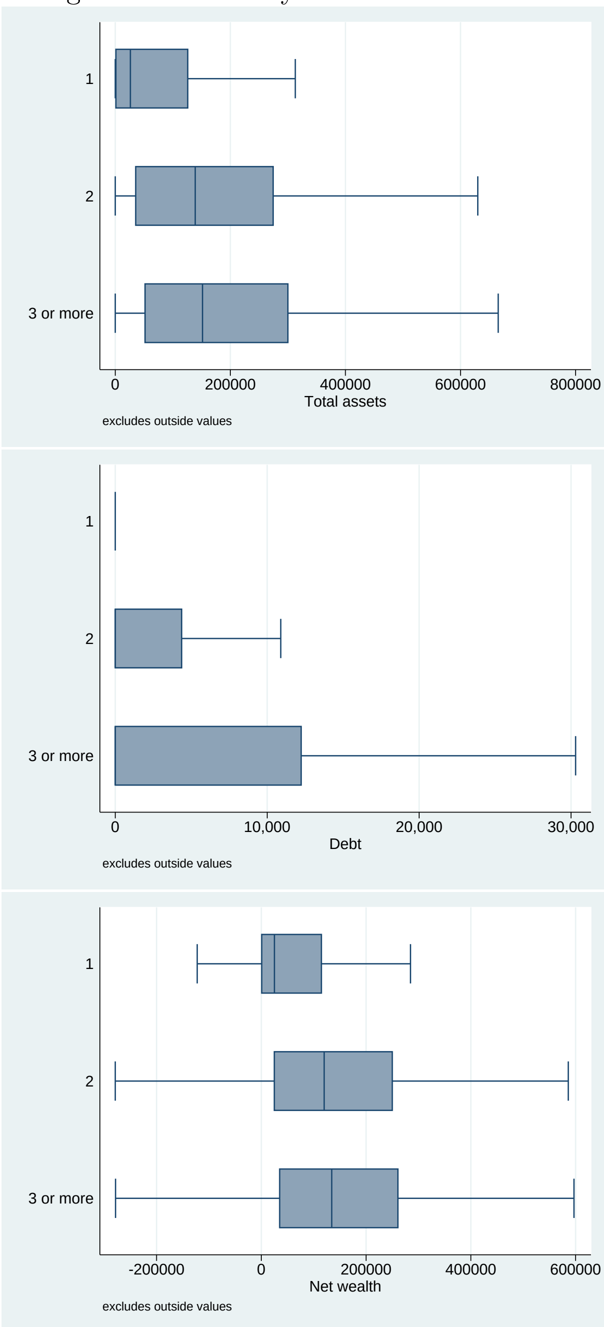
Source: SHARE data set, first wave. Figures in 2004 euros. The statistics for each variable are computed with the help of the maximal sample, giving missing observations. It follows that sample size varies from one column to the next. These computations do not rely on SHARE survey imputations (on this question, see Börsch-Supan and Jürges (2005b)).

We now turn to the overall distribution of real property values, total assets, debt and net worth. Net worth, or net wealth, is defined as the sum of all assets, corporate shares and real estate minus debts. The vast majority of wealth values are nonnegative. The histograms of these empirical distributions are given by Fig. 9, Fig. 10, Fig. 11 and Fig. 12, respectively. On each of these figures, the left-hand-side plot represents the distribution of the variable (which is always very skewed and highly dispersed, due to wealth inequality), while the right-hand side presents the distribution of the logarithm of the same variable, in the subsample where observations of the variable are not zero. The right-hand side plots show that the distribution of the logarithm of nonzero observations can reasonably be approximated by a bell-shaped density. The distribution of financial assets is dwarfed by the distribution of real estate assets. Debt too plays a minor role. Another view of the same empirical fact is provided by Figure 13. On this figure, the horizontal axis represents the percentiles of the distribution of a variable (*i.e.*, total assets, net worth and financial assets). The vertical axis reports the natural logarithm of the same variable. Fig. 13 clearly shows that debt is negligible, since the net worth and total asset inverse cdfs are very close to each other, while financial assets represent a minor share of total assets.

Next, we study the distribution of income.[1] Income is defined as the sum of labor income, pensions, asset income and income from real estate. The means are driven by extreme observations: it follows that the medians are more informative. Figure 14 gives the boxplots of the household's total income by country, ranked with respect to the medians, in ascending order. To interpret the figure, the reader must recall that this is the distribution of income among seniors aged 50 and more. As a consequence, Fig. 14 shows that the countries in which the seniors have more generous incomes and pensions: Scandinavian countries, the Netherlands and Switzerland, and to a lesser extent, France. In contrast, the median senior households are relatively poorer in Belgium, Germany, and the south of Europe. Fig. 15 plots the mean total income of a household as a function of age (*i.e.*, the principal respondent's age). We see that total income strongly decreases with age. Fig. 16 presents the distribution of income (upper plot) and the distribution of the log-income for nonzero observations (lower plot). This figure shows a bimodal distribution of the log-income, which looks like a mixture of bell-shaped distribution. A glance at Fig. 14 show that there are two groups of countries, the senior friendly countries (Switzerland, Netherlands, Sweden, Denmark, France) and the rest (Spain, Italy, Israel, Austria, Germany, Greece, Belgium). In each of these two subsets taken separately, the distribution of log-income would not be markedly bimodal. Finally, Fig. 17 gives the inverse cdf of log-income, ranging from very low incomes ($5 = \ln(150)$) to 60% of the sample below ($10 = \ln(22,000)$) in the middle and topped with $12.5 = \ln(268,000)$, with even higher incomes in the last 5 percentiles. We are now able to assess the importance of bequests and gifts received relative to household income and wealth. Table 10 compares total income and total transfers in different European countries. Figure 18

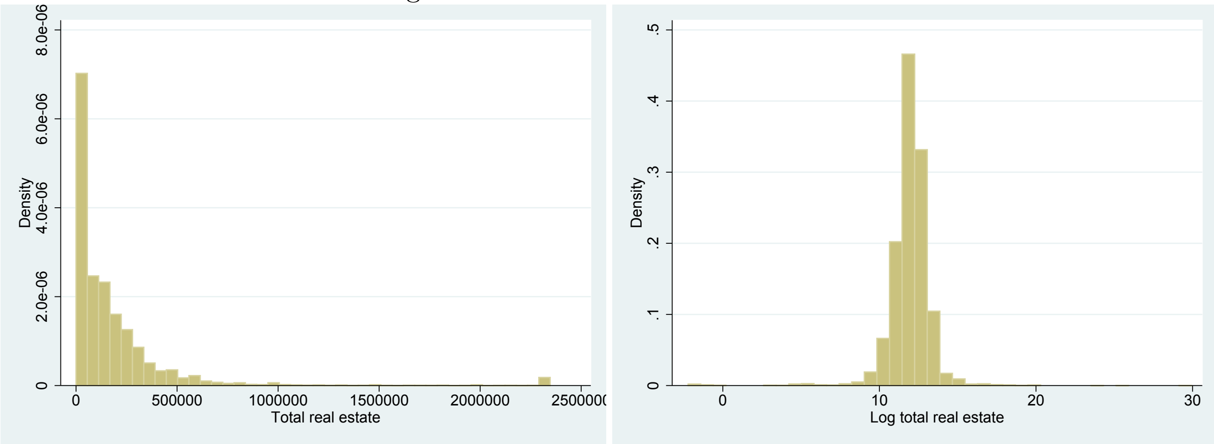
[1] There are some problems with missing data and many mistaken answers, particularly in the case of income. We first observed that missing data (on income and wealth) is significantly correlated with age but that this correlation is very weak. To be more precise, an additional year induces a 0.0005 increase in the probability of non-response for income. The corresponding figure is 0.001 per additional year of life in the case of wealth. We find that non-response on income and wealth is also correlated with gender. Females have a higher non-response rate; the probability of missing observation is 0.01 points higher than that of males. There is no detectable correlation of non-response with the country of origin.

Figure 8: Wealth by size of the household



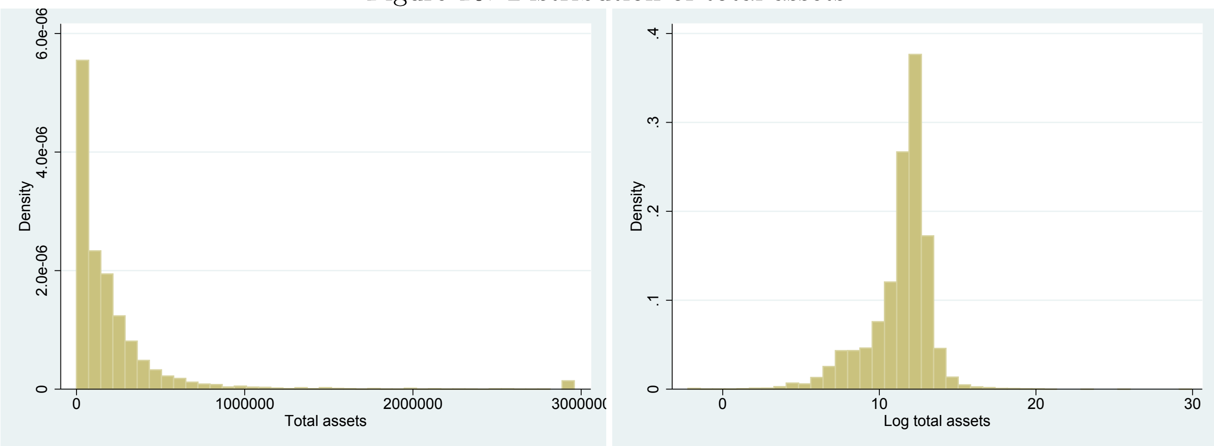
Source: SHARE data set, first wave. Figures in 2004 euros.

Figure 9: Distribution of real estate



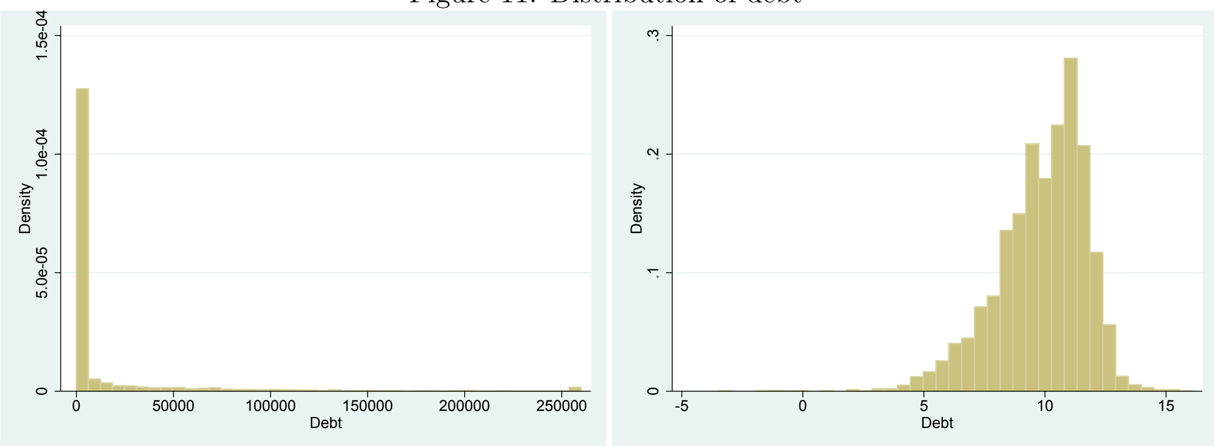
Source: SHARE data set, first wave.

Figure 10: Distribution of total assets



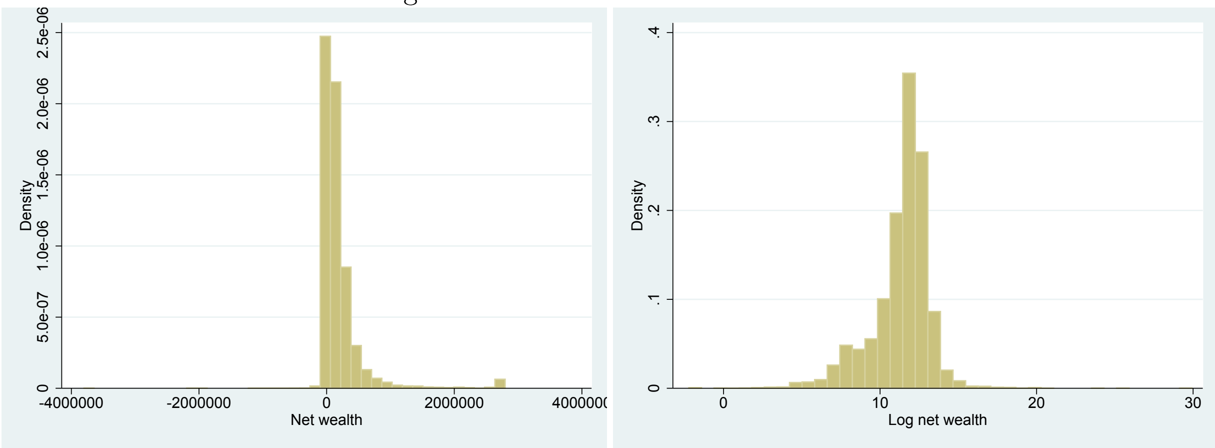
Source: SHARE data set, first wave.

Figure 11: Distribution of debt



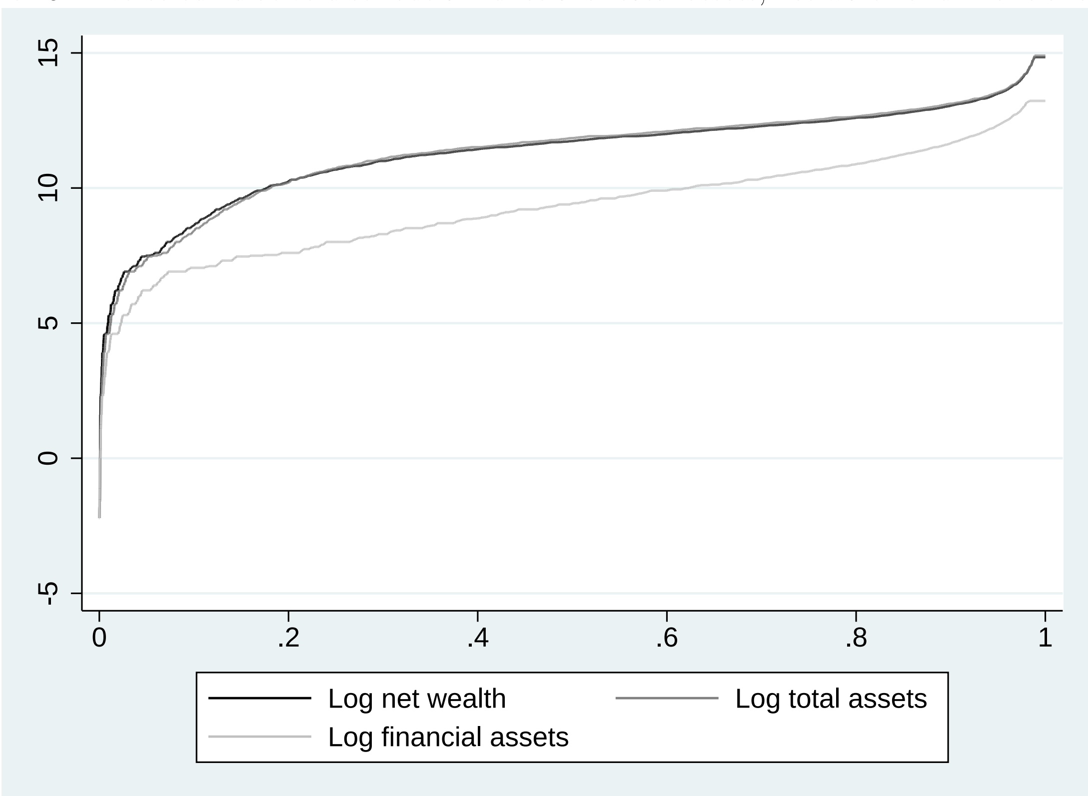
Source: SHARE data set, first wave.

Figure 12: Distribution of net worth



Source: SHARE data set, first wave.

Figure 13: Inverse cumulative distribution functions: total assets, net worth and financial assets



SHARE data set, first wave. The horizontal axis represents the percentiles of the distribution of a variable (*i.e.*, total assets, net worth and financial assets). The vertical axis reports the natural logarithm of the variable.

shows the ratio transfer/income in two groups of countries (northern vs. nouthern countries). The southern countries are Italy, Spain, Greece, Israel and France. These statistics have been computed for the subsample of households who received a positive transfer (and a positive income). The median for this ratio is 1.60 in northern countries and 1.62 in southern countries (the means are much higher, due to skewness and inequality and typically non-robust). Transfers are therefore substantial in the sense that the median ratio indicates a bequest representing 1.6 years of income. The same exercise can be applied to the ratio transfers/net worth. This is shown by Fig. 19. The median ratio is 0.28 in the northern countries and 0.22 in southern countries. Again, this is non-negligible. We consider the sub-population of individuals who reported both a positive amount of wealth and a positive income (10747 individuals), and we study the time profile of wealth as a function of age and income, to test for the presence of asset decumulation. The OLS regressions presented in Table 12 are of the following form,

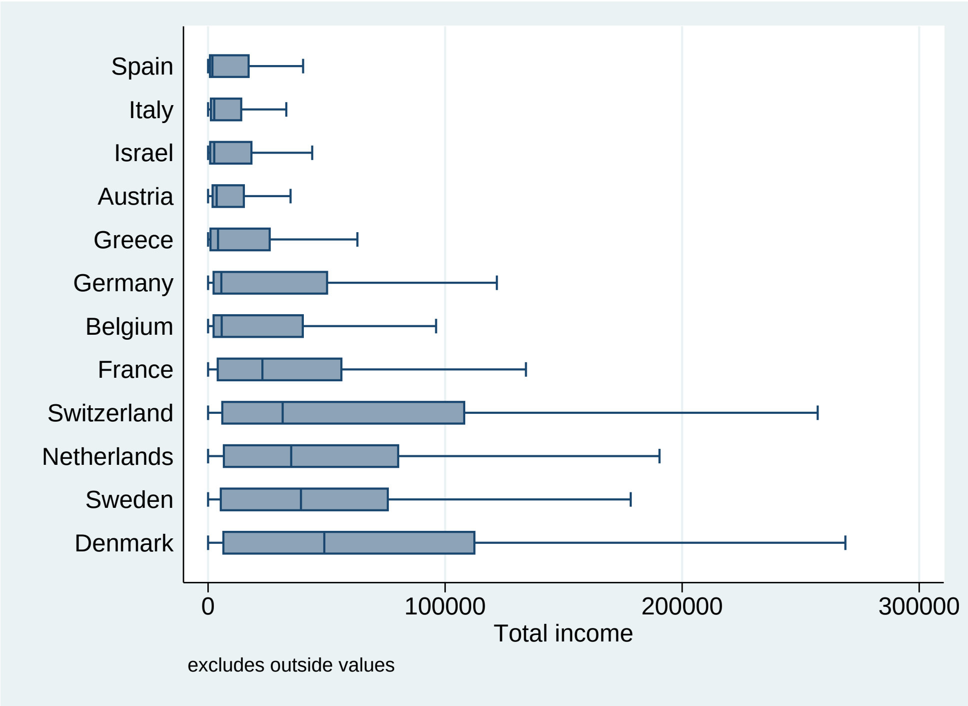
$$\log(\text{Wealth}) = \alpha\text{Age} + \beta(\text{Age})^2 + \gamma\log(\text{Income}) + \text{Controls} + \text{Random error}.$$

It is striking that we find significant and precisely estimated values of α and β and γ, all significantly different from zero at the 1% level. The maximum of wealth is reached when age is equal to $-\alpha/2\beta$. In Table 12, column (1), we find the simplest regression of total wealth on age, age squared and income, and we find the striking (and reassuring) result that $-\alpha/2\beta = 64$. This finding is relatively robust. In column (2) of Table 12, we introduced dummies for the so-called southern countries (France, Italy, Spain and Greece) and interactions of the coefficients on age and age squared with the southern-country indicator. The only significant thing that we find is that net wealth is smaller in these countries, but the wealth decumulation age remains above to 62. The computations of the decumulation threshold age are given by Table 11. If we break down wealth in financial assets and real estate, results are similar. The simplest regression of financial wealth on age and income yields a decumulation age $-\alpha/2\beta = 71$. Results are more or less the same if we add controls, in Table 12, column (4).

Regressions of real-estate on income, age and age-squared yield a maximum at age 64 (Table 12, column (5)) or 67 with controls for southern countries (Table 12, column (6)). See also Table 11. Further robustness tests should be performed, but it seems that some form of asset decumulation is taking place among persons aged 65 and more. If this is indeed the case, decumulation is non-negligible since the estimated values of β imply a decrease in wealth around 13% during the 10 years between ages 65 and 75.

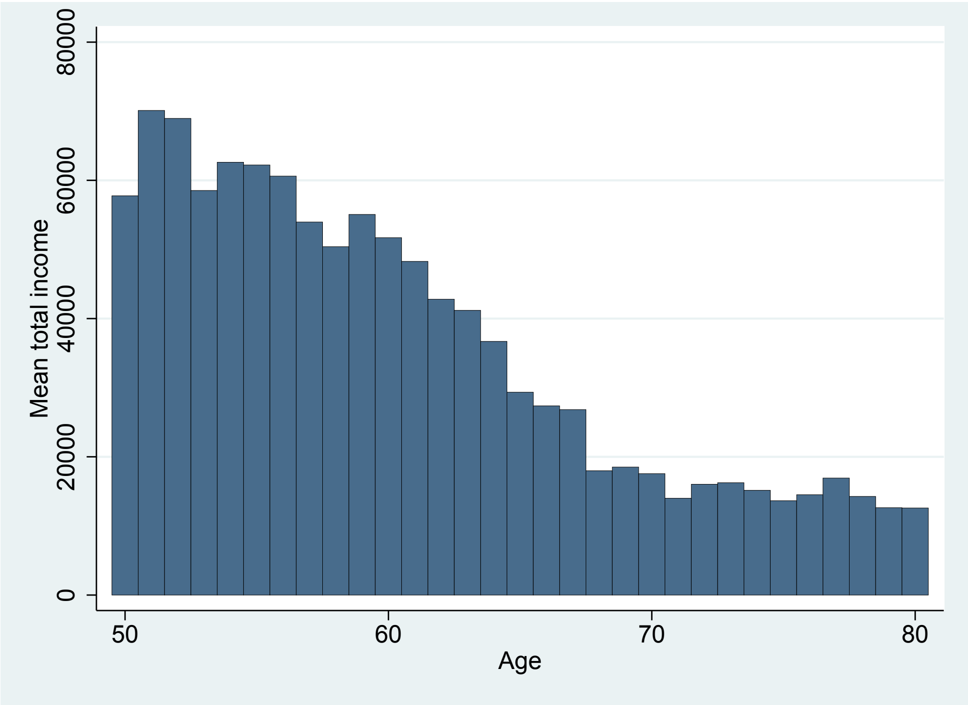
It may be that some cohort effects lead to a certain degree of overestimation (or underestimation) of the decumulation effect. But this bias is likely to be limited. This point deserves further investigation, but it is easy to see why the bias is likely to be small. Assume for simplicity that the above regression model applies to a panel of individuals. In a regime of balanced growth, income and assets would all grow at the same rate, say ρ. Let x denote the age of an individual. The assets and incomes of older individuals should be inflated to correct for the possible cohort effects. Assume that the assets of a person of age x are chosen to be $A(x) = Ae^{\rho x}$ (instead of A)

Figure 14: Total income by country



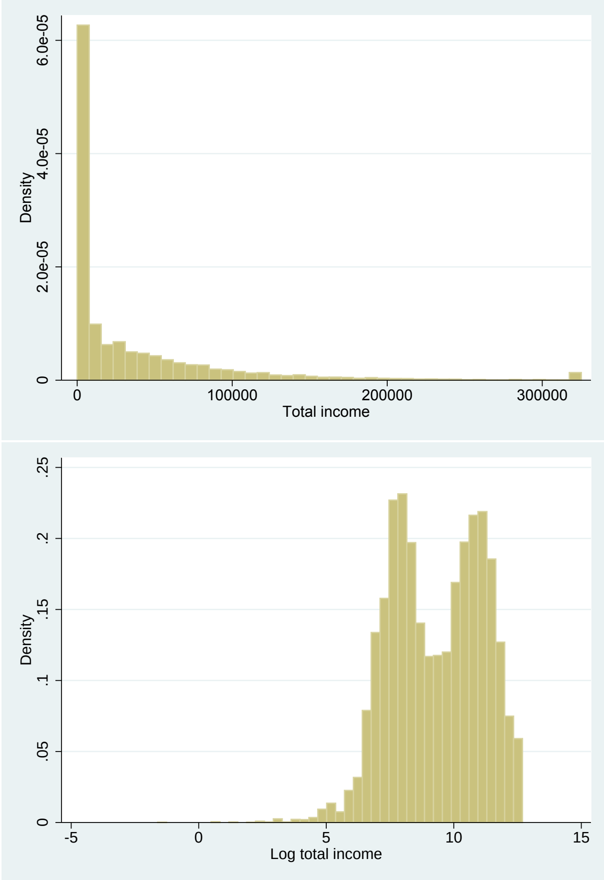
Source: SHARE data set, first wave.

Figure 15: Mean total income by age



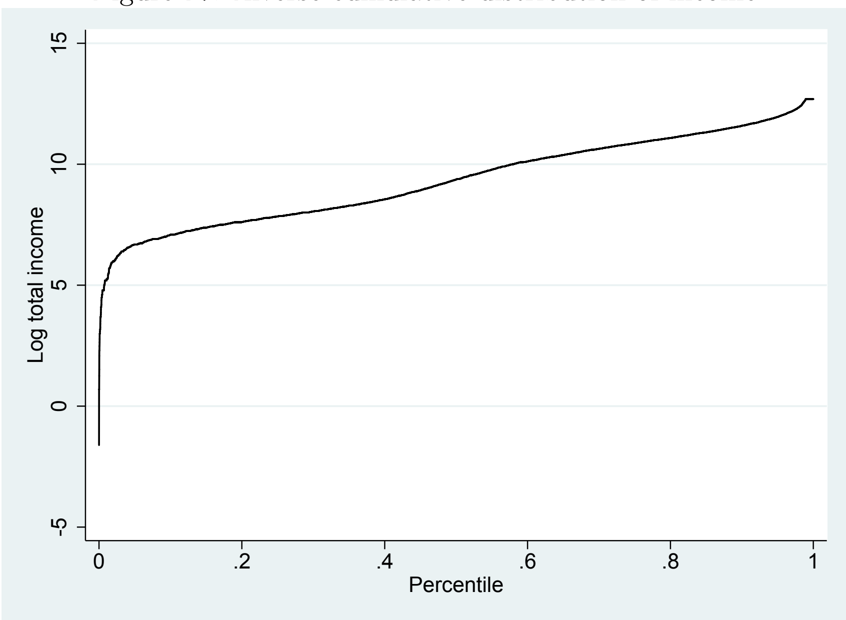
Source: SHARE data set, first wave.

Figure 16: Distribution of income



Source: SHARE data set, first wave.

Figure 17: Inverse cumulative distribution of income



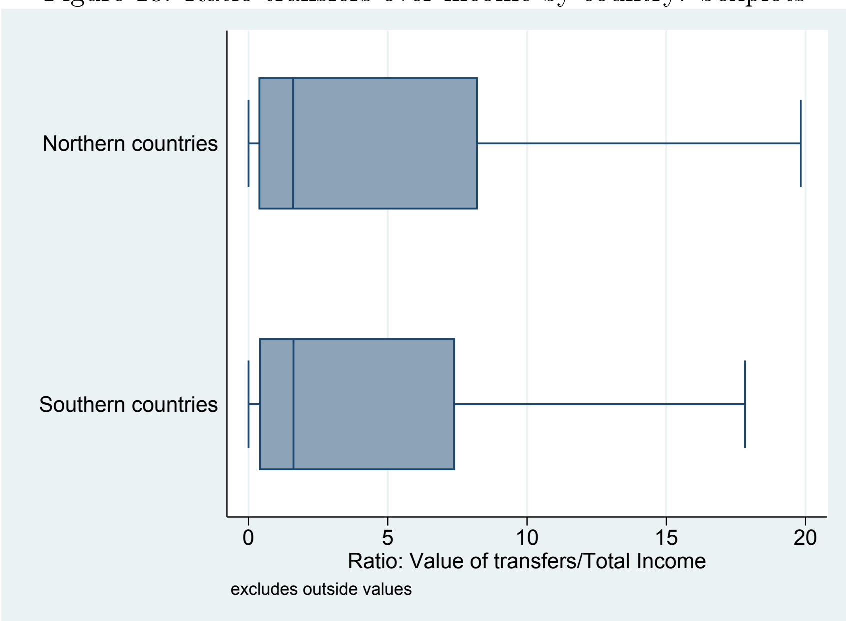
Source: SHARE data set, first wave.

Table 10: Income and transfers by country

	Total income		Received transfers	
	Mean	Median	Mean	Median
Austria	16,930	3,623	143,436	69,466
Germany	39,135	5,600	115,142	59,208
Sweden	51,402	39,186	48,773	25,257
Netherlands	53,855	35,060	66,325	29,811
Spain	17,249	1,840	58,326	23,584
Italy	16,501	2,600	84,365	39,734
France	43,468	22,926	103,283	47,956
Denmark	73,724	49,027	54,714	26,109
Greece	19,096	4,200	84,578	41,321
Switzerland	66,898	31,465	186,189	90,675
Belgium	31,584	5,735	94,183	47,034
Israel	19,815	2,603	78,867	42,993
Total	36,210	8,884	86,318	37,884

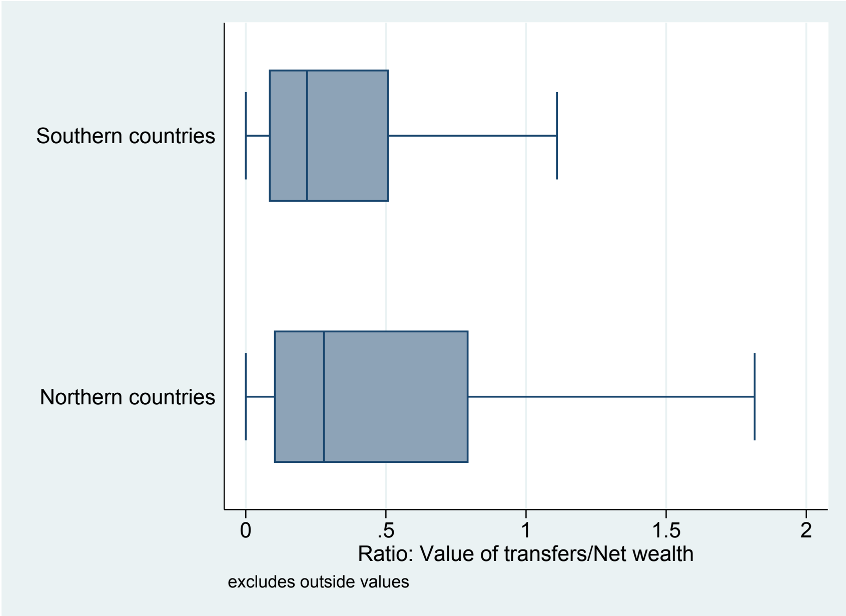
Source: SHARE data set, first wave.

Figure 18: Ratio transfers over income by country: boxplots



Source: SHARE data set, first wave.

Figure 19: Ratio transfers over net worth by country: boxplots



Source: SHARE data set, first wave.

and the income is $y(x) = ye^{\rho x}$ instead of y. Then, the above regression model becomes,

$$\log(A) = (\alpha_0 + (\gamma - 1)\rho)x + \beta x^2 + \gamma \log(y) + \text{controls} + \text{random error}.$$

Table 12, column (1), yields $\alpha = 0.11$, $\gamma = 0.2$. Take for instance $\rho = 0.03$ (a reasonable value for a real growth rate). Then, we find

$$\alpha_0 = \alpha + (1 - \gamma)\rho = 0.11 + 0.024 = 0.134$$

and the coefficient β does not change. The decumulation age would then be

$$-\alpha_0/2\beta = 0.134/0.0018 = 74.$$

This back-of-an-envelope calculation shows that the real α may in fact be slightly underestimated.

If α is only slightly biased, as suggested, we found that decumulation is non-negligible, since the estimated values of β imply a decrease in wealth around 13% during the 10 years between ages 65 and 75. It may be that the speed of rational dissaving, should be higher when survival probabilities are taken into account, but risk aversion and imperfect insurance may explain the observed facts, to a large extent. Given that the regressions presented in Table 12 are nonlinear in age, we computed a number of marginal effects drawn from regressions with effects varying by country and with interactions. These marginal effects are displayed in Table 13.

A further robustness test is provided by regressions of log-wealth on age, age squared, log-income, marital status and health status indicators. These regressions are displayed in Table 14. Health dummies are clearly very significant in the 3 columns of Table 14, with the expected signs and ranking of the magnitudes. Given that we control for age, a worse health status is correlated with smaller assets. Table 16 shows that decumulation starting around 65 is a robust finding.

Table 11: Age at which estate starts decreasing

	Net worth	Fin. Assets	Real Estate
All sample	63	71	64
Northern countries	62	71	64
Southern countries	67	70	67

Source: SHARE data set, first wave.

We conclude this preliminary empirical analysis of our topic with a study of the role of transfers in the building up of assets and real property. This is done by means of simple OLS regressions again. We estimated regressions of log-wealth on age, age squared, log-income and the logarithm of transfers, to estimate the impact of transfers on wealth accumulation. These regressions are displayed on Table 15. Interestingly, we find a significant, nonnegligible elasticity of assets with respect to transfers around .3 in the three regressions (*i.e.*, columns) of this table. Table 16 gives the estimated age at which three measures of wealth (net worth, financial assets and real property) start to decrease, on average.

Finally, we estimated the role of transfers in access to real property (mainly, the owner-occupier status). This is done by means of a linear probability model, regressing the ownership indicator on log-transfers and age, age squared and controls. Table 15 displays the result of three variants of this regression. Take for instance column (2) in Table 15. In this variant we control for age, age squared and age at first transfer. The profile of the probability of access to ownership is clearly a concave (quadratic) function of age, with a maximal probability at age 40. Given this, the probability of becoming an owner-occupier decreases further with the age of the principal respondent at the moment of the first transfer. Since the log-transfer itself has a significant and positive coefficient, we conclude that large and early transfers play a significant role in access to ownership, more or less as expected.

We now explore the possibility of improving efficiency in the transmission of income and wealth between parents and heirs/children, taking into account the risks associated with the longevity of parents. This is made possible, in particular by housing equity release instruments.

Table 12: Net wealth and assets explained by income, age and heterogeneity by country

	(1)	(2)	(3)	(4)	(5)	(6)
Dependent variable:	Net wealth	Net wealth	Fin. assets	Fin. assets	Real estate	Real estate
Age	0.1155***	0.1184***	0.1292***	0.0906***	0.0388**	0.0770***
	(0.0205)	(0.0259)	(0.0230)	(0.0284)	(0.0165)	(0.0217)
Age × Age	-0.0009***	-0.0010***	-0.0009***	-0.0006***	-0.0003**	-0.0006***
	(0.0001)	(0.0002)	(0.0002)	(0.0002)	(0.0001)	(0.0002)
Log total income	0.2257***	0.2681***	0.4327***	0.4151***	0.1234***	
	(0.0107)	(0.0143)	(0.0121)	(0.0157)	(0.0083)	
Southern countries		0.8459		-4.0363**		2.9262**
		(1.4762)		(1.6717)		(1.1594)
Southern countries × Age		-0.0071		0.1039**		-0.0897***
		(0.0420)		(0.0477)		(0.0334)
Southern countries × Age × Age		0.0001		-0.0007**		0.0007***
		(0.0003)		(0.0003)		(0.0002)
Southern countries × Log total income		-0.0686***		0.0078		0.1151***
		(0.0216)		(0.0247)		(0.0122)
Northern countries × Log total income						0.1280***
						(0.0115)
Constant	5.6743***	5.1505***	0.7644	2.4200**	9.4976***	8.2639***
	(0.7206)	(0.9147)	(0.8060)	(0.9990)	(0.5719)	(0.7501)
R^2	0.06	0.07	0.14	0.16	0.03	0.03
Number of Observations	10,747	10,747	8,414	8,414	9,991	9,991

Source: SHARE data set, first wave.
Significance: *** p-value<0.001, ** p-value<0.05, * p-value<0.1.
Method: OLS

Table 13: Marginal effects for the regressions with heterogeneous effect by country

	Net wealth	Fin. assets	Real estate
Age	-0.0048**	0.0081***	-0.0004
	(0.0020)	(0.0022)	(0.0017)
Southern countries	0.2979***	-0.4323***	-0.0247
	(0.0355)	(0.0410)	(0.0275)
Log total income	0.2413***	0.4179***	0.1225***
	(0.0108)	(0.0122)	(0.0084)
Number of observations	10,747	8,414	9,991

Source: SHARE data set, first wave.
Significance: *** p-value<0.001, ** p-value<0.05, * p-value<0.1.

3 Existing Institutions and Comparative Study

The present section is devoted to the description of a number of institutions that are relevant to understand the real-world context of our formal analysis. In the following, subsection 3.1 is devoted to equity release instruments. The latter are important building blocks of our analysis. Subsection 3.2 is devoted to taxation of inter-vivos gifts and inheritance in Europe.

3.1 Housing equity release contracts: home reversion, lifetime mortgages and *viager*

As noted in the introduction, it is well-known that many aging households keep the whole or the most important part of their wealth locked in their homes. Real estate prices going up in many urban areas have made many owner-occupiers more wealthy than they expected to be. Various forms of equity release contracts could be used, in many countries, to increase the income and pensions of the seniors. It is also well-known that the recourse to these contracts is still limited, for various reasons. Insufficiently developed financial markets, the tax system and behavioral problems are among the main reasons for the slow growth of these financial markets. Another important reason for this is just the fact that these markets have been opened recently. In the United States, the reverse mortgage programs started in 1989: this is relatively recent. It may simply be that households have not yet fully learned about these products and their use, because learning about these matters takes a generation or two. In continental Europe, the underdevelopment of home equity release markets may also be due to regulation, which is sometimes very strict. In general, these products have a bad reputation: they are viewed as "dangerous" by regulators.

The UK, Canada and Australia have a relatively more developed equity release activity. In the UK for instance, there exits several important types of equity release contracts: the lifetime mortgage and the home reversion. The *lifetime mortgage* is a loan secured on the borrower's home. The loan is repaid by selling the property after the borrower's death. The borrower retains

Table 14: Net wealth and assets explained by income, marital status and health

	(1)	(2)	(3)
	Net wealth	Fin. assets	Real estate
Age	0.0930***	0.1110***	0.0322*
	(0.0204)	(0.0228)	(0.0166)
Age × Age	-0.0007***	-0.0007***	-0.0002*
	(0.0001)	(0.0002)	(0.0001)
Log total income	0.1877***	0.3879***	0.1119***
	(0.0109)	(0.0123)	(0.0085)
Health indicators	(Reference:	very good)	
good	-0.1131**	-0.1774***	-0.0415
	(0.0549)	(0.0618)	(0.0412)
fair	-0.3607***	-0.4391***	-0.1120***
	(0.0546)	(0.0616)	(0.0416)
bad	-0.4725***	-0.7664***	-0.1079**
	(0.0642)	(0.0726)	(0.0502)
very bad	-0.4842***	-0.9066***	-0.2565***
	(0.0909)	(0.1056)	(0.0726)
Widowed=1	-0.5711***	-0.4653***	-0.1790***
	(0.0503)	(0.0564)	(0.0418)
Constant	6.8220***	1.9144**	9.8416***
	(0.7194)	(0.8007)	(0.5754)
R^2	0.08	0.17	0.03
Number of observations	10,739	8,409	9,985

Source: SHARE data set, first wave.
Significance: *** p-value<0.001, ** p-value<0.05, * p-value<0.1.
Method: OLS.

Table 15: Net wealth and assets explained by gifts

	(1) Net wealth	(2) Fin. assets	(3) Real estate
Age	0.0783**	0.1029**	0.0541*
	(0.0316)	(0.0411)	(0.0316)
Age × Age	-0.0006***	-0.0007**	-0.0004*
	(0.0002)	(0.0003)	(0.0002)
Log total income	0.2068***	0.3886***	0.1169***
	(0.0154)	(0.0206)	(0.0145)
Log total transfers	0.3406***	0.2755***	0.3285***
	(0.0199)	(0.0261)	(0.0183)
Constant	3.7923***	-0.3111	5.7925***
	(1.1023)	(1.4407)	(1.0809)
R^2	0.15	0.17	0.12
Number of observations	3,048	2,462	3,155

Source: SHARE data set, first wave.
Significance: *** p-value<0.001, ** p-value<0.05, * p-value<0.1.
Method: OLS.

Table 16: Age at which wealth or assets start decreasing for regressions with health or transfers

	Net worth	Fin. Assets	Real Estate
with health	68	79	64
with transfers	64	74	62

Source: SHARE data set, first wave.

Table 17: Ownership explained by gifts

	(1)	(2)	(3)
	Owner	Owner	Owner
Log total transfer	0.0403***	0.0395***	
	(0.0052)	(0.0057)	
Age	0.0193**	0.0162*	0.0189**
	(0.0085)	(0.0093)	(0.0088)
Age × Age	-0.0002***	-0.0002**	-0.0002***
	(0.0001)	(0.0001)	(0.0001)
Log total income	-0.0035	-0.0016	-0.0018
	(0.0041)	(0.0045)	(0.0042)
Gender	-0.0769***	-0.0899***	-0.0973***
	(0.0130)	(0.0143)	(0.0134)
Age at first transfer		-0.0017***	-0.0023***
		(0.0006)	(0.0006)
Constant	0.0499	0.2011	0.5462*
	(0.2990)	(0.3253)	(0.3037)
R^2	0.04	0.05	0.03
Number of observations	4,022	3,472	3,914

Source: SHARE data set, first wave. Linear probability model.
Significance: *** p-value<0.001, ** p-value<0.05, * p-value<0.1.
Method: OLS.

ownership of the house. Some of the value of the property can be ring-fenced as an inheritance for children. Borrowers can choose to make repayments, to pay only interest, or to let the accrued interest 'roll-up'. The lifetime mortgage may embody several elements of insurance: caps on the interest rate (if the rate is not fixed) and most importantly, there is a *no negative equity guarantee*. According to this guarantee, when the property has been sold, if the amount is not enough to repay the loan to the lender, the estate (*i.e.*, the heirs or the borrower) will not be liable for the difference between sales revenue and total debt. The latter provision entails an element of life insurance (because of the longevity risk of the borrower) combined with insurance of the real-estate market risks (*i.e.*, house-price risk) — this is a key reason for the existence of a minimum borrower age. The borrower has the right to remain in the property for life, or until he or she needs to move to long-term care. The released money can be drawn as a lump sum or in smaller amounts through time, or both.

Under the *home reversion* plan, part or all of the person's home is sold to a specialized intermediary in return for a lump sum or regular payments. The seller has the right to continue living in the property until death, rent free, but agrees to maintain and insure the property. Again a percentage of property can be ring-fenced to form a bequest. At the end of the home reversion plan, the property is sold by the intermediary and the proceeds are shared according to the remaining proportions of ownership. The no negative equity guarantee applies. Again, a home reversion plan embodies an element of life insurance: the provider bears the seller's longevity risk and the future house price risks.

In the United States and Canada, there exists a closely related contract called the *reverse mortgage*. Again, the contract allows elders to access equity built up in their homes and defer payment until after death. Interest is added to the loan balance each month and can grow to exceed the value of the home, but the estate is generally not required to repay any loan balance.

France has a traditional and original form of contract closely related to home reversion called the *viager* contract. This form of contract is based on the breakdown of property rights in French civil law. The property of an asset, and in particular, that of a house, can be decomposed in three parts: *usus* (*i.e.*, the right to use the good), *fructus* (*i.e.*, fruit: the right to earn the asset's income or rents) and *abusus* (the right to sell or dispose of the good). The *bare property* is by definition the ownership of a house, without usus and fructus, the latter being the rights to inhabit and (or) earn the associated rents. Under the viager contract, the buyer pays a lump sum, called *bouquet*, and commits to pay a sum of money every year or month, called *rent*, to the seller until the seller dies. The buyer is the *bare owner* of the house until the seller's death; retrieves full ownership when the seller passes away. The seller becomes the usufructuary (that is, is entitled to usus and fructus) when the viager is signed, for life. The buyer is clearly bearing the seller's longevity risk. The value of the house at the moment of the seller's death is also uncertain. Bare ownership means that the property is not yielding any rents. It follows that the discounted, expected (actuarial) value of the periodical payments of the seller cannot be greater than the discounted expected value of the future rents that can be earned after the seller's death. In France, most of the viager contracts are signed between individuals: until recently, intermediaries played a negligible role in this market. The market is underdeveloped because buyers must bear the longevity risk of the seller, and households cannot diversify this risk. In addition, there is a risk of default on the part of the buyer. The volume of viager sales could grow if powerful, deep-pocket intermediaries did invest a sufficient amount of capital in this market, because intermediated viager contracts could render the same services as home reversion plans. The usefulness of this type of contract may of course depend on the tax treatment of bare ownership vs. that of usus and fructus. Bare ownership can be profitable in France because it is exempt of the tax on capital called ISF. The home reversion plan and the French viager are very close in economic terms. Under a home reversion contract, the seller can stay in the home as a tenant, rent free, for life. In the viager contract the usufructuary is like a rent-free tenant. The usufructuary can in principle let the house to earn rents, but many contracts specify that the buyer's full ownership is retrieved if the seller moves to another residence. Under the latter contactual clause, home reversion and viager are almost equivalent (up to tax-treatment differences).

3.2 Taxation of Bequests and Gifts in Europe

We will focus here on inheritance (or estate) taxes on the one hand, and on *inter-vivos* transfer (or gift) taxes on the other hand. We'll use the European Union as an example, to illustrate the diversity of modes of taxation of bequests and gifts. Inheritance and gift taxes are taxes on transfers of wealth. In a way they are capital taxes; this is particularly obvious when the tax applies to real property and land, but also when a firm is transferred from on generation to the next.

Taxation of Inheritance

Inheritance is taxed widely in Europe. To be precise, it is taxed in 18 out of the 28 EU member states. Austria, Sweden and Portugal are notable exceptions. Inheritance taxes in Europe have a common structure: they are levied on the market value of a net inheritance; the taxpayer is the heir; most inheritance taxes are progressive; they discriminate according to family ties (close relatives and spouses typically enjoy lower rates and(or) higher exemptions); the surviving child or spouse is often fully exempt from paying the tax; there are special regimes for business asset transfers in most national tax systems.

The treatment of close relatives has a substantial influence on the effective tax rates. Most EU states recognize the freedom of bequeathing to close relatives to a certain extent; the most common justification being that people should be able to take care of their relatives after death (within limits). So, most small bequests are largely untaxed, and it should not be surprising to find that inheritance taxes contribute a small proportion of GDP in most countries. Belgium is the country in the EU with the highest relative importance of inheritance and gift taxation as a share of total tax revenue; the Belgian figure is 1.37%. The equivalent figure for France, that comes second, is 0.97%. The revenues of inheritance and gift taxes in the EU are everywhere below 1% as a share of GDP. It is also well-known that the inheritance tax is everywhere hotly debated (the "death tax"). The political economy of this type of taxation leads to a form of instability of its provisions. Since the year 2000, five European countries have abolished the tax completely (including Austria, Italy and Slovakia).

The tax structures in the 18 countries taxing bequests have the following common features. Taxes are almost always general, meaning that the tax base is the entire net estate. The taxpayer is the heir. In Denmark and the UK, the estate itself is taxed (inheritance tax vs. estate tax), but this distinction is secondary for the purpose of the present study. The tax rate has a structure with different scales: a different schedule applies to close relatives. The more distant is the heir, the higher the rates. The taxes are progressive (only Denmark, Italy and Croatia use a flat rate in the EU), but the degree of progressiveness varies substantially from one country to the other. The lowest rates are 6% on average. For more distant heirs, the rate is 20% on average. France, Germany, Ireland and Spain are special because the rate on children can be as high as 30%. Third parties are in some countries taxed at very high rates (80% in Belgium and Spain). The most common exemptions apply to small bequests and to family members: spouses and children receive large exemptions; 11 out of 18 countries fully exempt the surviving spouse; only 7 out of the 18 countries fully exempt the children. The transmission of family businesses poses a specific problem. It follows that 12 out of the 18 countries taxing bequests have a business continuity exemption, mostly in the form of a reduction in the value of the assets subject to the tax.

To compute effective tax rates, it is necessary to construct illustrative cases. We consider three such cases: a) the modest bequest (a house worth 135,000 euros and savings equal to 33,750 euros); b) the house is worth 338,000 euros on average and the value of other assets is equal to 169,000 euro; c) the wealthy parent case, a house valued 540,000 euros, savings amounting to 135,000 and shares in a business worth 675,000 euros. We assume in turn that the heir is a spouse (S) , a child (C), or a distant (non-related) person (D). The average effective inheritance tax rate in the EU is 2.8% in case aC; 5.75% in case bC and 5.12% in case cC. France stands out in Europe because of high tax rates on children with rates 7%, 15.7% and 12.3% in cases aC, bC and cC, respectively. Denmark, the Netherlands, Belgium and Finland are similar in this respect. The other countries have very low or zero effective rates when the heir is a child. The treatment of spouses is different; the average effective rates are 0.89%, 1.7% and 1.7% in cases aS, bS and cS, respectively. Finland, Belgium (Walloon Region) and Poland stand out with taxation rates around 7%, but most EU countries have rates close or equal to zero for spouses. In contrast, rates are on average around 20% in the (D) cases. They may be very high for non-related heirs in countries like France or Belgium, about 60%. These figures are based on official EU data (cf. European Commission (2014)).

Taxation of Gifts

Let us now consider gift taxes. Most EU countries have a gift tax or a provision for gifts in the income tax. Austria, Portugal, Sweden and a number of eastern European countries (Baltic states, Romania) are notable exceptions. Most countries have exemptions and tax the beneficiary of gifts at a rate that depends on family relationships. Effective tax rates can be assessed using the same illustrative cases as in the subsection devoted to inheritance taxes. The average effective gift tax rate in the EU is 3% in case aC; 5% in case bC and 4% in case cC. France stands out in Europe because of high tax rates on children with rates 7%, 15.7% and 12.3% in cases aC, bC and cC, respectively. Denmark, the Netherlands, Belgium and Finland are similar in this respect with effective gift tax rates around 14%. The other countries have very low or zero effective rates when the heir is a child. The treatment of spouses is different; the average effective gift tax rates are 2.5%, 3.5% and 3% in cases aS, bS and cS, respectively. Finland, France, Belgium (Walloon Region) and the Netherlands stand out with gift taxation rates around 15%, but most EU countries have rates close or equal to zero for spouses. In contrast, rates are on average around 20% in the (D) cases. They may be very high for non-related heirs in countries like France or Belgium, about 60%. The zero-rate countries, like Sweden, are now an exception.

Gift taxes are a small share of a country's GDP; yet, they may be an important source of tax revenues for some local governments and municipalities. Another important problem is tax evasion. If the donor wants to transmit the ownership of a particular dwelling or a piece of land, a notary will record the transaction and a gift tax can always be charged. The transmission of shares of stock or shares of ownership in a firm may be costly to hide. The most common form of evasion is therefore the under-valuation of the transmitted assets. But for the vast majority of people, if the goal of parents is simply to help children, not to transmit a particular house or a particular firm, then, parents can give money in cash to their children, without any possibility of detection. Parents make gifts and often try to share their estate between their children during their lifetime. It follows that inheritance and gifts are imperfect substitutes, but gifts clearly dominate inheritance, in particular because of time preference and risk aversion.

A key question is that, insofar as they take the form of real estate, assets are illiquid for the vast majority of the people, and it may be difficult to give part of one's house to a child. In French civil law, the ownership rights can be broken down in several parts: usus, fructus and abusus (defined above) and as a consequence, the bare ownership (abusus) can be given to a child, but this gift will be taxed. However, fiscal optimization may require that the mother sells the bare ownership to her daughter. The latter's gift tax will be based on the value of bare ownership only (which is of course lower than the value of full ownership). The full ownership will be recovered by the daughter when her mother passes away, and the inheritance of the usus-fructus part of the mother's house will not be taxed. The interplay of gift taxation and inheritance taxation may therefore be rather subtle.

4 A Simple Model of Intergenerational Transmission of Wealth

We now describe and analyze a simplified model of family contracts, allowing the study of intergenerational transmission. The basic model has three periods and three agents: the mother, the daughter and the banker-insurer. The mother owns a house and is entitled to a stream of income. The daughter wishes to buy a home but, possibly, she doesn't have enough assets or cash to buy a house that is spacious enough. The mother would like to give something to her daughter: to a certain degree at least, the mother is an altruist. At the same time, the mother wants to stay in her house; she doesn't want to move or downsize her consumption of housing. The banker offers to sign a contract with the mother and the daughter simultaneously. Each of the two contracts can be seen as independent, involving the signature of the banker and one client only, but the banker agrees to sign each of the contracts only of the other contract is signed in the appropriate terms. Alternatively, one can view the two contracts as a single agreement between three parties. There is no difference between the two views at least as long as contract renegotiation is not considered. If renegotiation may happen, then of course, it is important to specify if unilateral renegotiations are permitted (with the bank) or if the agreement of more than two parties is needed to renegotiate. We are not considering renegotiation here.

In the simplest version of the model there is a single source of risk, namely, the risk affecting the duration of the mother's life. There are three periods and the mother may pass away at the end of the first period or survive, and pass away at the end of the second period. The daughter lives for three periods for sure. The mothers' after-tax financial and housing wealth is entirely transmitted as a bequest to the daughter at the beginning of the period immediately following her death. In the simplified version of the model, the house prices, the rents and the income of both the mother and the daughter are non-random. We focus on the life insurance problem, on the intergenerational transfer and housing-equity release problems.

The banker-insurer offers a reverse mortgage of a special kind, or a French *viager* contract to the mother and simultaneously, offers a mortgage contract to the daughter to buy her home. The bundle of contracts simultaneously allows for *(i)*, equity release (transforming an illiquid asset into cash, or a sequence of cash payments); *(ii)*, an inter vivos gift to the daughter; *(iii)*, an ordinary bank loan to the daughter, *(iv)* income annuitization for the mother; and *(v)*, independently of the existence of a bank loan, a certain amount of consumption smoothing through income insurance for the daughter.

4.1 Basic assumptions

We start with a three-period, discrete-time model with contingent goods. There are three agents, the mother, the daughter and the banker. The mother dies at the end of period $t = 1$ with probability $(1 - p)$ or survives until the end of period $t = 2$ with probability p. We assume $0 < p < 1$.

There exists a competitive "spot" market and a rental market for housing in each period $t = 1, 2, 3, ...$ The spot price of a square meter in period t is denoted q_t. The rent of a square meter available in period t is denoted R_t. We assume that these prices are non-random and that the agents have perfect foresight, for simplicity. We will see later how these assumptions can be relaxed and how restrictive they are.

The mother owns a house of size $H_0 > 0$. House size is expressed in units of physical area, *i.e.*, in square meters. The daughter buys a house of size H_1; she may also rent square meters in period t.

We now define contingent consumption. There are two goods in the economy at each period: a consumption good and housing services. The mother's consumption in period 1 is denoted c_{01}. If the mother survives, her consumption in period 2 is denoted c_{02}. The consumption profile of the daughter if the mother survives in period 2 is denoted (c_{11}, c_{12}, c_{13}). If the mother dies in period 1, the continuation consumption profile, from $t = 2$ on, is denoted $(\hat{c}_{12}, \hat{c}_{13})$.

The daughter dies at the beginning of period $t = 4$; she sells her house on a futures market in period $t = 3$. The daughter has no children and no bequest motive, and as a consequence, she liquidates her housing capital entirely before death. For simplicity, the daughter doesn't adjust ownership after period 1: H_1 is fixed in periods $t = 1, 2, 3$.

The daughter can also rent additional square meters at each date $t = 1, 2, 3$. Again we consider contingent housing consumption. Let h_t denote rented housing in square meters if the mother survives in period 2 and let $\hat{h}_t$ denote rented square meters in periods $t = 2, 3$ if the mother passes away at the end of period 1.

The mother can sell a fraction of her property to the bank and keep the *usus* and *fructus*, that is, the mother remains a tenant until death, paying a zero rent; *i.e.*, she signs a French *viager* contract, or a home reversion contract on a fraction of her house. Let z_0 denote the fraction of the house sold at $t = 1$ as a *viager* contract. The mother can also downsize her housing consumption and sell a fraction z_t, $t = 1, 2$ on the spot market in periods $t = 1$ and $t = 2$ if she survives. We assume that $z_t \geq 0$ and obviously, the mother cannot sell more than her house,

$$0 \leq z_0 + z_1 + z_2 \leq H_0.$$

The mother and the daughter are endowed with additively separable, instantaneous Von Neumann-Morgenstern utility functions for consumption and housing. Let $u(c)$ denote the instantaneous utility of consumption (the same for the mother and the daughter, to keep the model simple). Let v_0, resp v_1 denote the separable utility of housing services for the mother and the daughter, respectively. Let β_0, resp. β_1 denote the discount factors of the mother and the daughter, respectively. The mother's instantaneous utility for housing consumption is denoted $v_0(h_{0t}; H_0)$, where h_{0t} represents current consumption and H_0 is the reference point, *i.e.*, the house owned by the mother at the beginning of period 1. We assume the following.

Assumption 1.
1. Utility v_0 is continuous, strictly increasing, strictly concave and differentiable almost everywhere with respect to current housing consumption h_{0t}. We assume the existence of a kink at point (H_0, H_0). The left-hand side partial derivative of v_0 with respect to the first variable h_{0t} is higher than its right-hand derivative.
2. The utilities u and v_1 are twice continuously differentiable, strictly increasing and strictly concave functions.

The assumption relative to v_0 allows us to formally represent the psychological fact that the mother clings to her house, as a kind of "habit formation" model for housing. The concavity assumptions formally capture risk aversion as usual.

Given these assumptions, the instantaneous utility of the mother in period $t = 1, 2$ can be written as follows,

$$v_0\left(H_0 - \Sigma_{\tau=1}^{t} z_\tau\right) + u(c_{0t}).$$

The expected-discounted utility of the mother is defined as follows,

$$U = v_0\left(H_0 - z_1\right) + u(c_{01}) + p\beta_0[v_0\left(H_0 - z_1 - z_2\right) + u(c_{02})] \tag{1}$$

The instantaneous utility of the daughter is given by the following expression,

$$v_1(H_1 + h_t) + u(c_{1t}).$$

The expected-discounted utility of the daughter can be written as follows,

$$\begin{aligned} V = v_1\left(H_1 + h_1\right) + u(c_{11}) & \\ + \sum_{t=2}^{3} \beta_1^{t-1}\left(pv_1\left(H_1 + h_t\right) + (1-p)v_1(H_1 + \hat{h}_t)\right) & \\ + \sum_{t=2}^{3} \beta_1^{t-1}\left(pu(c_{1t}) + (1-p)u(\hat{c}_{1t})\right). & \end{aligned} \tag{2}$$

We now consider the financial contracts and the taxes.

1. A lump sum (*i.e.*, the *bouquet*), denoted K is paid by the bank to the mother at the beginning of period $t = 1$.

2. The mother receives an annuity from the bank in period $t = 2$, denoted M_0, *if she survives.*

3. The net transfer from the bank to the daughter is denoted M_t, $t = 1, 2, 3$ if the mother survives in period 2, and denoted $\hat{M}_t$, $t = 2, 3$ if the mother passes away at the end of period 1.

4. Let x_t denote the transfer from the mother to the daughter (a gift of the mother in period t is a nonnegative x_t).

Let τ_g denote the tax rate on inter vivos transfers, and let τ_b denote the tax rate on bequests. Let y_{0t}, resp. y_{1t}, denote the income of the mother, resp., of the daughter in period t.

4.2 Actuarial budget constraints

To understand how the model works, we first write the agent's budget constraints. The mother's budget constraint in period $t = 1$ is defined as follows,

$$c_{01} + x_1 \leq K + y_{01} + q_1 z_1.$$

The mother can use the *bouquet* K and the revenue from downsizing $q_1 z_1$ to fund consumption and a gift x_1. The daughter's budget constraint is

$$c_{11} + R_1 h_1 + q_1 H_1 \leq M_1 + y_{11} + (1 - \tau_g)x_1.$$

We can aggregate the two budget constraints and obtain the family's budget constraint in period $t = 1$, that is,

$$c_{01} + c_{11} + R_1 h_1 + q_1 H_1 + \tau_g x_1 \leq K + M_1 + y_1 + q_1 z_1,$$

where by definition, $y_t = y_{0t} + y_{1t}$. The mother's budget in period $t = 2$, if she survives, is as follows,

$$c_{02} + x_2 \leq M_0 + y_{01} + q_2 z_2.$$

The daughter's period-2 budget constraint, if the mother survives, is the following,

$$c_{12} + R_2 h_2 \leq M_2 + y_{12} + (1 - \tau_g) x_2.$$

The aggregate family constraint in period 2 is then,

$$c_{02} + c_{12} + R_2 h_2 + \tau_g x_2 \leq M_0 + M_2 + y_2 + q_2 z_2.$$

The daughter's budget constraint in period $t = 2$, if the mother dies in period $t = 1$, is given by the expression,

$$\hat{c}_{12} + R_2 \hat{h}_2 \leq \hat{M}_2 + y_{12} + q_2 (H_0 - z_0 - z_1)(1 - \tau_b),$$

where the pre-tax bequest is given by $q_2(H_0 - z_0 - z_1)$.

The daughter's budget constraint in period $t = 3$ if the mother dies in period $t = 1$ is the following inequality,

$$\hat{c}_{13} + R_3 \hat{h}_3 \leq \hat{M}_3 + y_{13} + \frac{q_4 H_1}{(1 + r)},$$

where r is the interest rate and

$$\frac{q_4 H_1}{(1 + r)}$$

is the present value of the daughter's house, sold at the discounted price $q_4/(1+r)$ but available only in period $t = 4$, after the daughter's death. We will see later the role played by the assumption that the daughter sells her home in the last period of her life. It is in fact the appropriate assumption in this finite horizon model. The assumption means that the daughter can consume the value of her house in period 3 as if she benefited from a reverse mortgage at rate r. Finally, the daughter's budget constraint in period 3, if the mother passed away in period 2, must be expressed as follows,

$$c_{13} + R_3 h_3 \leq M_3 + y_{13} + \frac{q_4 H_1}{(1 + r)} + (1 - \tau_b) q_3 (H_0 - z_0 - z_1 - z_2),$$

where

$$q_3 (H_0 - z_0 - z_1 - z_2)$$

is the pre-tax value of the bequest in this case.

We now write the banker's intertemporal actuarial constraint. The banker can freely borrow or lend at interest rate r. The banker invests $L = M_1 + K$ at the beginning of period 1 and obtains a benefit B if the mother dies in $t = 1$ or $\hat{B}$ if the mother dies in $t = 2$. These benefits are expressed in present value terms, discounted back in period 1.

If the mother dies in period $t = 2$, the banker's intertemporal budget constraint can be written,

$$B - L = \frac{q_3 z_0}{(1+r)^2} - M_1 - K - \frac{(M_0 + M_2)}{(1+r)} - \frac{M_3}{(1+r)^2},$$

where

$$\frac{q_3 z_0}{(1+r)^2}$$

is the value of the viager sale, given that the mother survives. This is because a fraction z_0 of the mother's property is sold by the bank at spot price q_3 in period 3. If the mother dies in period $t = 1$, the banker's intertemporal budget constraint can be written,

$$\hat{B} - L = \frac{q_2 z_0}{(1+r)} - M_1 - K - \frac{\hat{M}_2}{(1+r)} - \frac{\hat{M}_3}{(1+r)^2},$$

where

$$\frac{q_2 z_0}{(1+r)}$$

is the value of the viager sale, given that the mother dies early, since a fraction z_0 of the house is sold at price q_2 in period 2. We assume perfect divisibility of the mother's property but we will check later that this assumption is harmless because the optimal choice happens to be $z_0 = H_0$. Given these expressions, the expected present value of the bank's profit is thus,

$$pB + (1-p)\hat{B} - L = Xz_0 - \frac{pM_0}{(1+r)} - K - M_1 - \sum_{t=2}^{3} \frac{(pM_t + (1-p)\hat{M}_t)}{(1+r)^{t-1}},$$

where by definition

$$X = (1-p)\frac{q_2}{(1+r)} + p\frac{q_3}{(1+r)^2}$$

is the value of a unit of the house sold under a *viager* contract in period 1.

Now, the expected profit of the bank is nonnegative if and only if the following actuarial constraint holds,

$$Xz_0 \geq K + M_1 + \frac{pM_0}{(1+r)} + \sum_{t=2}^{3} \frac{(pM_t + (1-p)\hat{M}_t)}{(1+r)^{t-1}}. \quad (3)$$

First define the expected-discounted value of family income Y. By definition,

$$Y = y_1 + \frac{(py_2 + (1-p)y_{12})}{(1+r)} + \frac{y_{13}}{(1+r)^2}.$$

We can now substitute the mother's and the daughter's budget constraints, expressed as equalities, in the above nonzero profit constraint of the bank. This yields the actuarial resource constraint, denoted (AC), in the economy under study. We easily obtain the following inequality,

$$\begin{aligned} & c_{01} + c_{11} + R_1 h_1 + \tau_g x_1 + q_1 H_1 \\ & + \frac{p}{(1+r)}(c_{02} + c_{12} + R_2 h_2 + \tau_g x_2) + \frac{p}{(1+r)^2}(c_{13} + R_3 h_3) \\ & + \frac{(1-p)}{(1+r)}(\hat{c}_{12} + R_2 \hat{h}_2) + \frac{(1-p)}{(1+r)^2}(\hat{c}_{13} + R_3 \hat{h}_3) \\ & \leq Xz_0\tau_b + (H_0 - z_1 - z_2)(1-\tau_b)X + q_1 z_1 + z_2 \frac{q_2}{(1+r)}(1 - \tau_b(1-p)) \\ & + Y + \frac{q_4 H_1}{(1+r)^3}. \end{aligned} \quad (4)$$

Remark that if the tax rate on bequests is zero, *i.e.*, $\tau_b = 0$, then the fraction of the house sold under a *viager* contract is irrelevant, *i.e.*, z_0 doesn't appear in the utility functions, and it doesn't appear in the resource constraint (4). The *viager* contract is therefore mainly used to avoid the inheritance tax. This is because the bank and the family members can freely arrange the profile of transfer payments in a contingent way.

4.3 Optimal family contracts

We can now study the optimal family contracts. Our main assumption is that the mother is a pure altruist, at least to a certain extent. This is formalized as follows.

Assumption 2. The preferences of the mother over contingent resource allocation in the three periods are represented by the utility $U+\gamma V$, where $\gamma > 0$ is a parameter measuring the mother's degree of concern for the welfare of her daughter.

We can now formulate a key remark. Maximizing $U + \gamma V$ subject to the actuarial constraint (4), the sign constraints, $c_{jt} \geq 0$, $h_t \geq 0$, $Z_t \geq 0$ and $H_0 \geq z_0 + z_1 + z_2$ yields a Pareto-optimal contract in the economy comprising the mother, the daughter and the banker. The set of Pareto-optimal arrangements is described when γ varies in the set of positive real numbers. A fully general optimization problem characterizing the Pareto-optima can be obtained if we rewrite the resource constraint as an expression for the bank's expected profit $\bar{B}$. It is easy to see that the bank's expected profit is just the RHS of (4) minus the left hand side of (4). So, the optimal contract problem can be reformulated as that of maximizing $U + \gamma V$ subject to $V \geq V_0$ and $\bar{B} \geq B_0$, the sign constraints, and $H_0 \geq z_0 + z_1 + z_2$, where V_0 and B_0 are parameters of the problem. These parameters are minimal values of the daughter's utility and the banker's expected profit, resp. Varying V_0 is equivalent to varying γ in the characterization exercise. In the present section, we focus on zero-profit optima, but first order necessary conditions would be the same if we choose a positive value of profit $\bar{B} > 0$.

Remark that the maximization of $U+\gamma V$ subject to (4) and sign constraints is a well-behaved convex programming problem. The objective is a concave function and the linear constraints clearly define a convex feasible set. It follows that the Kuhn and Tucker first-order necessary conditions for optimality are also sufficient. These conditions are studied in the Appendix. Now, define the left derivative of v_0

$$\frac{\partial v_0(h, H_0)}{\partial h}^{-} = \lim_{h \to H_0, h < H_0} \frac{v_0(h, H_0) - v_0(H_0, H_0)}{h - H_0}.$$

The value of this left derivative at point (H_0, H_0) represents the strength of the mother's psychological ties to her house. It may also represent other unmodeled reasons for willing to remain an owner occupier. The following proposition is proved in the appendix.

Proposition 1. The mother's house is sold entirely under a *viager* contract if the tax rate on inheritance is positive. If

$$\frac{\partial v_0(H_0, H_0)}{\partial h}^{-}$$

is large enough and if $\tau_b > 0$, then, any optimal contract satisfies

$$z_1^* = z_2^* = 0$$

and

$$z_0^* = H_0,$$

where starred variables denote optimal values.

For proof, see the Appendix.

We now consider the daughter's choice of house size H_1^* and the daughter's tenure choice. The daughter is allowed to rent and (or) to buy a certain quantity of housing. We want to characterize the market conditions under which the daughter indeed at least weakly prefers to buy her home over renting. We obtain the following proposition.

Proposition 2.

1. The daughter wishes to buy her house if the following arbitrage inequality is satisfied,

$$q_1 \leq R_1 + \frac{R_2}{(1+r)} + \frac{R_3}{(1+r)^2} + \frac{q_4}{(1+r)^3} \tag{5}$$

2. If the daughter rents a positive quantity of housing, *i.e.*, $\hat{h}_t^* > 0$ and $h_t^* > 0$ then, the above arbitrage inequality holds as an equality.

3. If the arbitrage inequality holds as a strict inequality and the daughter doesn't rent a positive quantity of housing, *i.e.*, $\hat{h}_t^* = 0$ and $h_t^* = 0$, then, the optimal size of her house, H_1^*, is given by the following equation,

$$v_1'(H_1^*)\left(1 + \beta_1 + \beta_1^2\right) = \frac{\lambda^*}{\gamma}\left(q_1 - \frac{q_4}{(1+r)^3}\right). \tag{6}$$

For proof, see the Appendix.

If the housing market is competitive and if the standard arbitrage equation linking rents and prices holds (the arbitrage inequality holds as an equality), renting and buying are equivalent decisions. In this case we could easily assume that the daughter has (at least) a slight preference for ownership and derive the result that renting is zero. This preference for ownership may be well-justified, for various reasons that are not captured by our simple model. A simple way of introducing a bias towards ownership is to assume that the daughter's utility for housing services v_1 has two arguments, *i.e.*, we have $v_1 = v_1(h, H)$ instead of $v_1 = v_1(h + H)$, and the marginal utility of a square meter is higher when owned. We focus on daughters with at least a slight preference for ownership.

The next step is to study the optimal consumption profile of the mother and the daughter: this yields indications on the consumption smoothing or income insurance properties of the optimal family contract. We obtain the following proposition.

Proposition 3. The optimal consumption profile satisfies the following properties. The actuarial resource constraint's Lagrange multiplier is positive, that is, $\lambda > 0$, and it is equal to the marginal utility of the mother's consumption in the first period,

$$u'(c^*_{01}) = \lambda. \tag{7}$$

We also find the sharing rules,

$$u'(c^*_{01}) = \gamma u'(c^*_{11}) \tag{8}$$

$$\beta_0 u'(c^*_{02}) = \gamma \beta_1 u'(c^*_{12}) \tag{9}$$

We find the intertemporal allocation rules,

$$\beta_0 u'(c^*_{02}) = \frac{u'(c^*_{01})}{(1+r)} \tag{10}$$

$$\beta_1 u'(c^*_{12}) = \frac{u'(c^*_{11})}{(1+r)} \tag{11}$$

$$\beta_1^2 u'(c^*_{13}) = \frac{u'(c^*_{11})}{(1+r)^2}, \tag{12}$$

and the *full insurance property*, that is,

$$c^*_{12} = \hat{c}^*_{12} \tag{13}$$

$$c^*_{13} = \hat{c}^*_{13}. \tag{14}$$

For proof, see the Appendix.

The sharing rules show how parameter γ, measuring the mother's altruism, determines the division of income between the two women. The intertemporal allocation rules are standard "Euler equations". The dynamics of consumption are determined by the discount factors β_1 and β_0. The optimal consumption is constant through time if $\beta_0 = \beta_1 = 1/(1+r)$. The last two properties express the fact that the daughter is completely insured against the consequences of the mother's risk of early death. These properties imply that the daughter's consumption profile doesn't depend on the event of the mother's early death. The banker is clearly providing the insurance here.

Finally, we determine the optimal gifts. We have the following clearcut result.

Proposition 4. If the tax rate on inter vivos gifts τ_g is positive, the optimal gifts are zero, that is, $x^*_t = 0$ for $t = 1, 2$.

For proof, see the Appendix.

The interpretation of the result is that, if the intermediated transfers between the mother and the daughter are not considered as straightforward gifts, then, these transfers coming from the bank are cheaper and should be preferred over any direct transfer of income (or property) between the mother and the daughter. The optimal family contract may be interpreted as a hidden transfer from the mother to the daughter by the tax authority. It is then possible to give a definition of the implicit transfers made through the bank intermediary, but to a large extent, this definition is a question of convention. We will discuss this point in greater detail below.

It is now possible, and easy, to compute the optimal family contract. The first thing that we need is the optimal value of λ^*. If λ^* is known, then, using the equations stated in Proposition 3, we derive the optimal consumption plan for the mother and the daughter. Finally, the optimal consumption plan in turn determines the optimal *bouquet* K^*, the optimal *annuity* M_0^*, the optimal *net transfers* between the bank and the daughter, M_t^* and the optimal size of the daughter's property H_1^*.

By assumption, the utility functions u and v_1 are invertible. Let ϕ denote the inverse of u and ψ denote the inverse of v_1. With these definitions, it is easy to solve for optimal consumption and housing as a function of λ^*. We have,

$$\begin{aligned}
c_{01}^* &= \phi(\lambda^*); \qquad c_{11}^* = \phi\left(\frac{\lambda^*}{\gamma}\right); \\
c_{02}^* &= \phi\left(\frac{\lambda^*}{(1+r)\beta_0}\right); \qquad c_{12}^* = \phi\left(\frac{\lambda^*}{\gamma(1+r)\beta_1}\right); \\
c_{13}^* &= \phi\left(\frac{\lambda^*}{\gamma(1+r)^2\beta_1^2}\right); \\
H_1^* &= \psi\left[\frac{\lambda^*(q_1 - q_4(1+r)^{-3})}{\gamma(1+\beta_1+\beta_1^2)}\right].
\end{aligned}$$

We then substitute these expressions in the actuarial resource constraint (4) above, expressed as an equality since $\lambda^* > 0$ implies that this constraint is always binding. This yields an equation in λ^* that can be solved in many standard cases. At the optimum contract, the expression for the actuarial resource constraint boils down to,

$$c_{01}^* + c_{11}^* + q_1 H_1^* + \frac{p c_{02}^*}{(1+r)} + \frac{c_{12}^*}{(1+r)} + \frac{c_{13}^*}{(1+r)^2} = H_0 X + Y + q_4 \frac{H_1^*}{(1+r)^3}. \tag{15}$$

The latter equation determines the value of λ^*. The solution is easy to find if utilities are homogeneous functions, for instance, with the usual CRRA form $u(x) = (x^{a+1} - 1)/(a+1)$.

Given the optimal values of c_{jt}^*, and H_1^* we finally derive the optimal contract payments as follows.

Proposition 5. The optimal intermediated family contract is defined by the following list of variables.

$$z_0^* = H_0; \tag{16}$$

$$z_1^* = z_2^* = 0; \tag{17}$$

$$K^* = c_{01}^* - y_{01}; \tag{18}$$

$$M_0^* = c_{02}^* - y_{02}; \tag{19}$$

$$M_1^* = c_{11}^* + q_1 H_1^* - y_{11}; \tag{20}$$

$$M_2^* = c_{12}^* - y_{12} = \hat{M}_2^*; \tag{21}$$

$$M_3^* = c_{13}^* - y_{13} - \frac{q_4 H_1^*}{(1+r)} = \hat{M}_3^*. \tag{22}$$

The optimal contract is characterized by the absence of inheritance, full housing equity release on the part of the mother, full annuitization of the mother's income, consumption smoothing for the daughter. The optimal contract typically entails a loan to the daughter, so that we expect $M_1^* > 0$ because the period 1 transfer to the daughter covers the price of the house $q_1 H_1^*$ and later transfers may be negative to repay the loan, principal and interest, that is, we would typically have $M_2^* < 0$ and $M_3^* < 0$.

5 Extensions and Generalizations

We can now study various kinds of extensions and generalizations of the basic model. There are several kinds of interesting extensions.

1. Multi-period extension of the basic model with a single heir.
2. Multi-person extensions of the model to accommodate several children.
3. Multiple-risks extensions of the basic model. The basic model treats the problem of life insurance. But other sources of risk are important, mainly:

   (a) The risk of a severe health problem, forcing the mother to move to a nursing or retirement home, and requiring additional resources (long-term care risk).

   (b) The income risk borne by the daughter. The daughter may become unable to face mortgage repayments, leading to foreclosure.

   (c) The risks affecting property prices and rents.

4. The banking monopoly model.

The problem may become difficult to analyze in its full generality, but we know that the four kinds of extensions listed above are feasible, using the same basic economic principles, at the cost of some simplifying assumptions, to keep the computations tractable. Point 4 in the above list boils down to considering profit maximization instead of utility maximization. The optimal contracts that maximize the banker's expected profit are easy to characterize: they basically share the same features of all optima, but a complete study of the bank monopoly problem requires a careful description of the family's *individual rationality* (or *participation*) *constraint.* We start with a sketch of the multiperiod extension.

5.1 Multiperiod Extension

Let the time horizon of the model be denoted $\mathcal{T}+1$. The daughter dies at the end of this period. The mother dies at the end of period T, which is random. We assume that $1 \leq T \leq \mathcal{T}$. The daughter will outlive her mother by at least one period. Let p_T be the unconditional probability of the mother dying at T. Thus, we have,

$$\sum_{T=1}^{\mathcal{T}} p_T = 1.$$

The cumulative distribution is denoted P_t, that is, by definition,

$$\Pr(T < t) = \sum_{T=1}^{t-1} p_T = P_t.$$

The survival function is denoted S_t. By definition again, we have,

$$S_t = \Pr(T \geq t) = 1 - P_t.$$

The conditional density, or hazard rate, is therefore,

$$\Pr(T = t \mid T \geq t) = \frac{p_t}{S_t}.$$

To pose the optimal-contract problem in a fully general and rigorous way, we consider again contingent goods. The family contract becomes contingent with respect to the mother's death event, *i.e.*, to the drawing of $T \in \{1, ..., \mathcal{T}\}$.

Let (c_{0t}, c_{1t}) denote the consumption allocation if the mother survives in period t. Let H_1 be the size of the daughter's house in square meters and let h_t be the rented housing, also in square meters, if the mother survives in period t.

If $t > T$, let $c_{1t}(T)$ denote the contingent consumption of the daughter in periods following the mother's death at T. Additional contingent rented space is denoted $h_t(T)$ for $t > T$.

The mother can still decide to sell a fraction z_0 of her house under a *viager* contract and she can still downsize her housing consumption at any period $t \leq T$. Let then z_t, $t > 0$ denote the fraction of the mother's property sold on the spot market at date t. We have the sign constraints $z_t \geq 0$ for all t and the constraint,

$$H_0 \geq \sum_{t=0}^{\mathcal{T}} z_t.$$

The mother's utility, conditional on death at date T, denoted U_T, can be written

$$U_T = \sum_{t=1}^{T} \beta_0^{t-1} \left[v_0 \left(H_0 - \sum_{\theta=1}^{t} z_\theta \right) + u(c_{0t}) \right]. \tag{23}$$

The mother's expected utility is simply

$$U = \sum_{T=1}^{\mathcal{T}} p_T U_T = \sum_{T=1}^{\mathcal{T}} p_T \sum_{t=1}^{T} \beta_0^{t-1} \mathcal{U}_t, \tag{24}$$

where, by definition, $\mathcal{U}_t = v_0(H_0 - \Sigma_{\theta=1}^t z_\theta) + u(c_{0t})$. We can easily rewrite this expression as follows,

$$U = \sum_{t=1}^{\mathcal{T}} \beta_0^{t-1} \mathcal{U}_t \sum_{T=t}^{\mathcal{T}} p_T,$$

and therefore we finally obtain,

$$U = \sum_{t=1}^{\mathcal{T}} \beta_0^{t-1} S_t \left(v_0(H_0 - \Sigma_{\theta=1}^t z_\theta) + u(c_{0t}) \right). \tag{25}$$

The daughter's utility, conditional on T is denoted V_T, and by definition, we have,

$$V_T = \sum_{t=1}^{T} \beta_1^{t-1} \left[v_1(H_1 + h_t) + u(c_{1t}) \right] + \sum_{t=T+1}^{\mathcal{T}+1} \beta_1^{t-1} \left[v_1(H_1 + h_t(T)) + u(c_{1t}(T)) \right], \tag{26}$$

and

$$V = \sum_{T=1}^{\mathcal{T}} p_T V_T. \tag{27}$$

Using the fact that $S_{\mathcal{T}+1} = 0$ we can rewrite V as follows,

$$\begin{aligned} V = \sum_{t=1}^{\mathcal{T}+1} \beta_1^{t-1} S_t \left[v_1(H_1 + h_t) + u(c_{1t}) \right] \\ + \sum_{T=1}^{\mathcal{T}} p_T \sum_{t=T+1}^{\mathcal{T}+1} \beta_1^{t-1} \left[v_1(H_1 + h_t(T)) + u(c_{1t}(T)) \right]. \end{aligned} \tag{28}$$

We assume as above that the mother is a pure altruist and wishes to maximize $U + \gamma V$, where $\gamma > 0$ is a parameter measuring the degree of altruism.

We now consider budget constraints and derive the actuarial resource constraint in its general form. Let (M_{jt}) denote the transfers from the banker to the mother ($j = 0$) and to the daughter ($j = 1$). The transfer to the mother stops after death, that is, $M_{0t} = 0$ if $t > T$. The lump sum called *bouquet* above is just $K = M_{01}$ here. Given these definitions, the mother's budget constraints are written as follows,

$$c_{0t} + x_t \leq M_{0t} + y_{0t} + q_t z_t, \tag{29}$$

for all $t \leq T$. The daughter's budget constraints, for $t \leq T$, are written as follows,

$$c_{1t} + R_t h_t + q_1 H_1 \delta_{1t} \leq M_{1t} + y_{1t} + (1 - \tau_g) x_t, \tag{30}$$

where $\delta_{1t} = 1$ if $t = 1$ and 0 otherwise. The family's aggregate budget constraint now becomes,

$$c_{0t} + c_{1t} + R_t h_t + q_1 H_1 \delta_{1t} + \tau_g x_t \leq M_t + y_t + q_t z_t, \tag{31}$$

again for $t \leq T$. To simplify notation, we denote

$$M_t = M_{0t} + M_{1t} \qquad \text{and} \qquad y_t = y_{0t} + y_{1t}.$$

After T, the bank's transfers to the daughter at t are contingent to T, denoted $M_t(T)$. The daughter's budget constraints have a different structure for $t > T$. If $t > T+1$ and $t < \mathcal{T}+1$, we have,

$$c_{1t} + R_t h_t(T) \leq M_t(T) + y_{1t}. \tag{32}$$

At time $t = T+1$ the daughter receives the mother's bequest and pays the inheritance tax. We have,

$$c_{1,T+1}(T) + R_{T+1} h_{T+1}(T) \leq M_{T+1}(T) + y_{1,T+1} + (1-\tau_b) q_{T+1} \left(H_0 - \Sigma_{t=0}^{T} z_t \right). \tag{33}$$

At time $t = \mathcal{T}+1$ the daughter's house is sold at price $q_{\mathcal{T}+2}/(1+r)$ and the daughter dies at the end of period $\mathcal{T}+1$. If $T < \mathcal{T}$ we have the final budget constraint,

$$c_{1,\mathcal{T}+1}(T) + R_{\mathcal{T}+1} h_{\mathcal{T}+1}(T) \leq M_{\mathcal{T}+1}(T) + y_{1,\mathcal{T}+1} + q_{\mathcal{T}+2} \frac{H_1}{(1+r)}. \tag{34}$$

The banker's expected profit, conditional on T, is denoted π_T. For $1 \leq T \leq \mathcal{T}$, we have,

$$\pi_T = \frac{q_{T+1} z_0}{(1+r)^{T-1}} - \sum_{t=1}^{T} \frac{M_t}{(1+r)^{t-1}} - \sum_{t=T+1}^{\mathcal{T}+1} \frac{M_t(T)}{(1+r)^{t-1}}. \tag{35}$$

The bank's expected profit is by definition,

$$\pi = \sum_T p_T \pi_T.$$

The bank's profitability constraint is simply

$$\pi \geq 0.$$

This can be rewritten,

$$\pi = z_0 X - \sum_{t=1}^{\mathcal{T}} S_t \frac{M_t}{(1+r)^{t-1}} - \sum_{t=1}^{\mathcal{T}} p_T \sum_{t=T+1}^{\mathcal{T}+1} \frac{M_t(T)}{(1+r)^{t-1}} \geq 0, \tag{36}$$

where the unit value of a *viager* contract is given by the following expression,

$$X = \sum_{T=1}^{\mathcal{T}} \frac{p_T q_{T+1}}{(1+r)^T}. \tag{37}$$

To simplify notation, define the discount factor

$$\beta = \frac{1}{1+r}.$$

Now, if we substitute the agent's budget constraints, expressed as equalities, in the profitability constraint $\pi \geq 0$, taking into account the fact that $S_1 = 1$ and $S_{\mathcal{T}+1} = 0$, some easy algebra yields the following actuarial resource constraint,

$$q_1 H_1 + \sum_{t=1}^{\mathcal{T}} \beta^{t-1} S_t (c_t + R_t h_t + \tau_g x_t) + \sum_{T=1}^{\mathcal{T}} p_T \sum_{t=T+1}^{\mathcal{T}+1} \beta^{t-1} (c_{1t}(T) + R_t h_t(T))$$
$$\leq z_0 X + Y + \sum_{t=1}^{\mathcal{T}} \beta^{t-1} S_t q_t z_t + (1-\tau_b) \sum_{T=1}^{\mathcal{T}} \beta^T p_T q_{T+1} \left(H_0 - \Sigma_{t=0}^{T} z_t \right) + q_{\mathcal{T}+2} \beta^{\mathcal{T}+1} H_1. \tag{38}$$

The left-hand side of the 38 is the sum of the values of all contingent goods, including gifts, the price of which depends on the tax rate τ_g The right-hand side of 38 is the sum of all resources in expected present value terms. More precisely

1. The first term is the value of the *viager* sale z_0X.

2. The second term is the family's expected wealth, derived from income flows, that is, by definition,
$$Y = \sum_{t=1}^{\mathcal{T}+1} \beta^{t-1}(S_t y_t + P_t y_{1t}).$$

3. The third term on the RHS, that is, $\Sigma_{t=1}^{\mathcal{T}} \beta^{t-1} S_t q_t z_t$, is the expected value of revenues related to planned property-downsizing sales.

4. The next to last term is the net expected after-tax value of inheritance,
$$(1-\tau_b)\sum_{T=1}^{\mathcal{T}} \beta^T p_T q_{T+1}\left(H_0 - \Sigma_{t=0}^{T} z_t\right).$$

5. The last term is the value of the daughter's house once resold after the daughter's death. It follows that the effective present price of property is
$$q_1 - q_{\mathcal{T}+2}\beta^{\mathcal{T}+1}.$$

It is now possible to study the optimal family contract in the multiperiod extension of our model. The results obtained are, in essence, the same as before. The optimal intermediated family contracts are obtained as solutions of the following program: maximize $U + \gamma V$ subject to the actuarial resource constraint (hereafter ARC), 38.

Let λ denote the Lagrange multiplier of the ARC. Let μ denote the Lagrange multiplier of the feasibility constraint $H_0 - \Sigma_{t=0}^{\mathcal{T}} z_t \geq 0$. Then, from the first-order necessary conditions for optimality, we find the equation,
$$\lambda \tau_b \sum_{T=1}^{\mathcal{T}} \beta^T p_T q_{T+1} = \mu. \tag{39}$$
Exactly as in the simplified version of the model, we can state a number of results characterizing the optimal contract. Proposition 6 gathers a number of claims, the proof of which is in the Appendix. We find that if the mother's marginal utility of housing is large enough, she will sell 100% of her house under a *viager* contract. The conditions under which the daughter prefers ownership at period $t = 1$ over renting are, in essence, the same as before. The optimal consumption satisfies properties similar to those stated as Proposition 5 above.

Proposition 6.
(i) If the tax rate on bequests τ_b, is positive, and if the mother's marginal utility of housing, v_0', is large enough at reference point H_0, then, any optimal arrangement satisfies,
$$z_0^* = H_0 \qquad \text{and} \qquad z_t^* = 0 \quad \text{all} \quad t > 0.$$

(ii) The daughter prefers ownership at $t = 1$ over renting during all subsequent periods if the following arbitrage inequality holds,

$$q_1 \leq \sum_{t=1}^{\mathcal{T}+1} \beta^{t-1} R_t + \beta^{\mathcal{T}+1} q_{\mathcal{T}+2}.$$

(iii) Assuming that the arbitrage inequality holds and $q_1 > q_{\mathcal{T}+2}\beta^{\mathcal{T}+1}$ (the first period price of a square meter is not too low with respect to the fundamental discounted value of a square meter), then, the daughter's house size is uniquely determined by the following equation,

$$v_1'(H_1^*) = \frac{\lambda^*}{\gamma} \frac{(1-\beta_1)}{(1-\beta_1^{\mathcal{T}+1})} \left(q_1 - q_{\mathcal{T}+2}\beta^{\mathcal{T}+1}\right). \tag{40}$$

(iv) The optimal consumption path is determined as follows.

1. The Lagrange multiplier λ^* is positive and

$$u'(c_{01}^*) = \lambda^*.$$

2. The sharing rule is determined by γ as follows,

$$\gamma u'(c_{11}^*) = u'(c_{01}^*).$$

3. The inter-temporal allocation of consumption is determined by the Euler equations,

$$u'(c_{jt}^*) = u'(c_{j1}^*) \left(\frac{\beta}{\beta_j}\right)^{t-1},$$

for all $j = 0, 1$ and all $t \leq j(\mathcal{T}+1) + (1-j)\mathcal{T}$.

4. The contingent consumption $c_{1t}(T)$ doesn't depend on p_T, provided that it is positive: we find $u'(c_{1t}^*(T)) = (\lambda/\gamma)(\beta/\beta_1)^{t-1}$ and therefore, given the Euler equation above, the *full insurance property* holds, that is,

$$c_{1t}^*(T) = c_{1t}^*$$

for all $t > T$.

5. Finally, optimal gifts are zero because there is a tax on gifts. Kuhn-Tucker conditions yield $\lambda^* \beta^{t-1} S_t \tau_g x_t^* = 0$. It follows that $\lambda^* \tau_g > 0$ implies $x_t^* = 0$ for all $t \leq T$.

To solve the model completely, we again need to compute the optimal value of λ. This can be done by first solving for the consumption profile as a function of λ. Then, the ARC provides an equation that determines λ^* — the entire solution depends on this key parameter. This equation is nonlinear and must be solved numerically, except in special cases. Going back to the period-by-period budget constraints for the mother and the daughter taken separately, we derive the optimal values of transfers with the bank, *i.e.*, the M_{jt}^* values.

The multi-period extension of our family contract model provides a tractable benchmark to study optimal arrangements with the bank.

5.2 Multi-person Extension

Our model can easily be adapted to study the case of several children, several heirs. Assume for instance that the mother has k children, $k > 1$. Let $\gamma_i > 0$ the weight of child i in the mother's utility function; let V_i be the expected utility of child i. The mother is altruist and wishes to maximize $U + \sum_i \gamma_i V_i$. Again, any maximum of this objective function subject to the bank's zero-profit constraint is a Pareto-optimum. The multi-person extension doesn't pose any particular problems. Using additional constraints, like equal-sharing constraints, any restrictions bearing on the children's shares of the mother's estate can be embodied: the model can take the legal inheritance rules into account.

5.3 Multiple-risks Extensions

The various sources of risk lead to further generalizations of the family contract model. We discuss in turn the rent and house-price risks; the risk of a severe health problem; the risks affecting income.

1. *Variability of rents and house prices.*

Until this point we assumed that $(q_t)_t$ and $(R_t)_t$ were perfectly predictable sequences. Given that we focus on solutions in which the mother and the children are owner-occupiers, the variability of rents doesn't play a direct rôle in the derivation of the solution. This variability, of course, would come into play if we studied a family model with variable tenure choice. Our benchmark contract includes a full *viager* sale of the mother's house, meaning that the mother is fully protected (by the bank) against fluctuations in the value of the estate. Thus, the variability of rents and house prices is mainly a problem for the bank. If we go back to 38 above, we see that the key variable is the assessment of the *viager* sale XH_0. Recall that X is the discounted and expected value of a square meter under a *viager* contract with the mother. To be more precise, with almost no other change in the model, X could be replaced with a more general definition,

$$X = \mathbb{E}_T \left[\beta^T \mathbb{E}_q \left(q_{T+1} \,|\, T, q_0 \right) \right],$$

where $\mathbb{E}$ is the mathematical expectation operator. The outer operation, *i.e.*, $\mathbb{E}_T$, requires only the knowledge of life tables and the age of the mother. This is routine work for actuaries. The other operation, that is, $\mathbb{E}_q \left(. \,|\, T, q_0 \right)$ is a more difficult econometric problem, a real-estate price-forecasting problem. There are models, estimation methods and data sources to perform this job. But like other asset-price forecasting models, the variance of predictions increases with the time horizon, and the length of the phase during which the bank pays an annuity to the mother may easily be longer than 10 years. It follows that we have a real technical difficulty for the bank. But as asset-value risks in general, they can be diversified away or shared, at least partially, with the help of financial markets and the cooperation of specialized financial partners. A well-developed market for the type of contracts under consideration provides a sequence of discount rates, taking the risk affecting future house prices into account. We conclude that the pricing formula for X (*i.e.*, (37)) can be replaced with a more sophisticated or more realistic expression.

More generally, the mother may wish to stay an owner-occupier because, *(i)*, owning her house is like a call option with a given exercise price on a given number of square meters in the future; and because, *(ii)*, owning a house also immunizes the owner-occupier against the variability of rents; finally, *(iii)* there may be various tax optimization, country-dependent reasons for which owning one's home is advantageous.

(i) Ownership doesn't insure the landlord against fluctuations in property prices, but directly secures access to a number of square meters, in a given city; thus, it secures access to a quantity of real housing service consumption. In addition, in spite of being relatively illiquid, home-ownership is at the same time a relatively flexible store of value that can be used in case of an accident or of severe health problems: downsizing can in these cases be a way of releasing cash to cover unexpected expenses. So, the value of home-ownership is not independent from the quality of the mother's health insurance (see below). Ownership is also flexible because, to a certain extent, it allows for mobility in geographical space: the mother can move and keep more or less the same consumption of housing services.

(ii) In a well-arbitraged property market, prices, rents and interest rates are related but the risk affecting rents can be considered in isolation. The variability of house prices depends on the variability of rents, interest rate shocks and irrational exuberance, that is, the possible emergence of a real estate bubble. Renters are exposed to the variability of rents, and this variability translates into a variability of real housing consumption as well as ordinary consumption, through the household's budget constraint. An *owner-occupier* (not a pure landlord) is typically insured against these risks, even if he(she) plans to move under some contingencies.

(iii) The tax regime may also change the motives for home-ownership. As compared to a renter, an owner-occupier typically pays a property tax (or a tax on real estate capital), but since imputed rents are not taxed, with equal incomes, the renter and the owner pay the same income tax. The owner-occupier may sometimes in addition deduct mortgage interest payments from taxable income. It is well-known that in some countries, housing policies strongly favor ownership in this manner. Note that a *viager* contract, insofar as it transfers property to the banker, leads to a change in the mother's wealth and real-estate capital: these changes may trigger some additional advantages, like avoiding the property and wealth taxes.

2. *Risk of severe health problems. Long-term care insurance.*

The risks affecting health are of course crucial in old age. It may be that the reason why the take up rate of equity release products such as reverse mortgages and annuities is low because seniors are not well insured against these risks. In particular, the risk of an accident generating the need for costly assistance at home or the need to move to a retirement (or nursing) home, must be taken into consideration. The seniors would not want to sell their home under a *viager* contract or release equity because their house is a store of value, a precautionary saving against this type of need. Typically, the mother's apartment could be rented to generate the additional income needed to pay for a nursing home. Full property would typically be transferred to the buyer (the bank) under a typical *viager* contract, when the mother leaves her house, for whatever reason.

This is true, *except if the* viager *contract is specified differently*: it would clearly be possible to add covenants according to which the mother can let her apartment to generate cash in case of departure to a nursing home. This possibility could explicitly be open until she passes away. Alternatively, the banker could provide an insurance against this type of health risk in addition to the *viager* contract. The bundle of liquidity, credit and insurance services offered to the family can clearly be extended to include some long-term care insurance.

As a first approximation, we can simply model these risks as an income shock (the nurse's salary has to be paid out of current income). The simplest possible extension of the three-period model presented above is to assume that if the mother survives in period 2, she also suffers an income shock, with a certain probability, before passing away at the end of period 2. The insurance against this risk would take the form of an adjustment of M_0, the annuity payment, contingent on survival. This payment would simply become contingent on survival *and* loss of autonomy. This risk can be priced easily and the actuarial resource constraint can be adjusted to take it into account. Thus, a multi-period extension of the model with an additional risk of loss of autonomy, taking the form of an income shock, poses no special technical problems.

3. *Income shocks and mortgage foreclosure.*

We now discuss the random income shocks affecting the daughter(s). In case of a drop in current income, the daughter may no longer be able to repay her debt to the bank. Because of severe moral hazard and adverse selection problems, temporary or permanent losses of income cannot be insured by the market. Yet the loans of a certain size, granted to certain categories of clients are insured, but only against disability and the occurrence of premature death. It is well-known that most ordinary home loans are secured by collateral, typically by the property itself. It is therefore reasonable to extend the model to capture the event of foreclosure. We can simply assume that the daughter incurs a severe loss of income with some probability at each period, and that in that event, the daughter's house is seized and sold by the bank. Again, this type of extension can be treated by the introduction of additional states of nature, by adjustments of the actuarial resource constraint, to deal with new events in which the mother survives (or not) and simultaneously, the daughter loses her income (or not), etc. Foreclosure has a present expected value appearing as an additional resource for the bank, while the daughter's net repayments to the bank must be discounted for the appropriate probability of repayment. This type of extension will not change the optimal solution in a radical way.

5.4 Banking Monopoly Model

The basic family contract model can be turned "upside down" to study surplus extraction by the banker. Instead of maximizing $U + \gamma V$ subject to, say $\pi \geq 0$, or $\pi \geq \pi_{\min}$ we study the profit maximization problem, that is, maximize the expected profit π, subject to $U + \gamma V \geq W_0$, where W_0 is the best outside option of the family. By definition, the best outside option of the family is what the family can achieve without any help from the bank, when the mother and the daughter are subjected to taxes, subjected to all budget constraints, if the only financial products that they can use are a savings account with rate r and an ordinary mortgage with rate ρ and duration $d = \mathcal{T} + 1$ and if they cannot borrow otherwise.

We first define what the daughter alone can achieve if she can only save money at rate r, and cannot borrow, apart from obtaining an ordinary mortgage loan with rate $\rho > r$ and duration d, and there are no gifts of the mother, *i.e.*, $x_t = 0$ for all t. Let $V_0(z_1, ..., z_{\mathcal{T}})$ denote this expected utility level. This is a function of (z_t), $t > 0$, since the daughter receives the mother's bequest and since this bequest, in turn, is determined by T and the sequence (z_t).

By definition, V_0 is the maximum of V subject to these constraints. Under these constraints, the daughter cannot insure herself against the risk of receiving the mother's bequest late, although she can borrow to buy a house under an ordinary mortgage contract, knowing that inheritance will help with the reimbursement of the loan at some random point in the future.

Given that $V_0(z)$ is known, the individual rationality level W_0 of the mother can be defined as the value of the following program:

Maximize $U + \gamma V$

subject to all the mother's and the daughter's budget constraints, subject to $V \geq V_0$, allowing for a savings account at rate r; no equity release; no borrowing, apart in the form of an ordinary mortgage; non zero transfers x_t at each date are permitted, but $K = M_t = M_t(T) = 0$.

We could of course consider variants of these problems, depending on the feasible sets of actions available to the mother and the daughter when they stand alone. The value created by the bank is due to equity release and the various kinds of insurance offered to the mother and the daughter, plus the tax savings, potentially. This surplus can be extracted by the bank. The bank can also exploit the fact that the mother has a high marginal value for housing consumption at the reference point H_0. We study the banking monopoly model in greater detail in the next section.

6 Housing, Welfare and Profitability: Calibration and Numerical Simulations

We now endeavour to study the model by means of micro-simulations. We first specify the model fully by choosing functional forms for utility functions. We then calibrate the model, using appropriate actuarial life tables and values of parameters that are viewed as acceptable in the literature on risk aversion. Next, we generate a random sample of families reflecting these parameters. We compute the optimal contract for each individual in the sample and compute the value of an outside option for each of these families as well. We compute the utilities and the profit of a bank underwriting the family contract. Finally, we study the properties of housing demand, consumption, expected utility and expected profit in the artificially generated sample of families.

The optimal contract under study is the full-insurance family contract, exactly as defined in sub-section 5.1 above, with two-members families (a mother and a daughter). A more delicate choice is the outside option, because there are many possible variants of this option. The results, and in particular the bank's expected profit per contract depend on the next best opportunity of the family. We could for instance assume that the outside option has all the consumption smoothing possibilities of the optimal contract but no insurance against the mother's longevity risk. This type of outside option can be clearly defined analytically but the numerical simulation problems posed by the study of this form of solution are complicated. We decided to study a less sophisticated, but equally appealing form of outside option.

6.1 Definition of the Outside Option: the Rule-of-Thumb Contract

The rule-of-thumb (hereafter, the RoT) contract is an arrangement with the bank defined as follows. 1° The daughter buys a house of size H_1 at date $t = 1$ with a mortgage from the bank at a rate $\rho > r$. The mortgage is a constant amortization loan that lasts during the daughter's entire life, for simplicity. 2° The daughter sells the house during the last period of her life and gets a constant annuity (at rate r) during her entire life as a consequence. This is to make sure that the total expected and discounted value available to the family as a whole is the same in the RoT and in the optimal contracts. 3° The daughter receives the bequest at date $T + 1$. Again, the mothers' house (of size H_0) is sold immediately after the mother's death and transformed by the bank into an annuity (at rate r) that pays a constant sum until the daughter passes away. This last assumption ensures a form of consumption smoothing that is not perfect, since the rate of consumption of the daughter, under the RoT contract, has an upward jump at date $T+1$, due to inheritance. 4° A rule-of-thumb is used by the banker to determine the size of the daughter's house, and therefore, the amount of the loan. The mortgage repayment must be equal to a constant fraction ϕ (say, $\phi = 1/4$) of the daughter's income y_1. This assumption entails the fact that in the RoT contract, the banker doesn't take the mother's income and wealth into account to determine the loan size. 5°. Finally, there are no gifts from the mother to the daughter. In the outside option formulation, the daughter does not save: she just reimburses the mortgage and consumes the rest of her income. For simplicity, there is no downpayment to buy the house (but a nonzero downpayment of the daughter could easily be added).

We summarize the above points formally as follows.

a) The daughter borrows to buy a house of size H_1 at rate $\rho > r$. Define the discount factor

$$\delta = \frac{1}{1+\rho},$$

and recall that $\beta = 1/(1 + r)$. The daughter's mortgage is repayed during the daughter's entire life with a constant amortizement annuity AH_1, where

$$A = \frac{q_1(1-\delta)}{(1-\delta^{\mathcal{T}+1})}.$$

b) The daughter resells the house on a futures market in the year $\mathcal{T} + 1$ at price $\beta H_0 q_{\mathcal{T}+2}$. The constant annuity with the same discounted value is denoted $A_1 H_1$ where

$$A_1 = \frac{q_{\mathcal{T}+2}\beta^{\mathcal{T}+1}(1-\beta)}{(1-\beta^{\mathcal{T}+1})}.$$

c) The after-tax annuity value of the bequest, $q_{T+1}H_0(1-\tau_b)$, paid between periods $T+1$ and $\mathcal{T}+1$, has the following expression,

$$A_{0t}(T)H_0 = (1-\tau_b)H_0 \frac{q_{T+1}(1-\beta)}{(1-\beta^{\mathcal{T}-T+1})},$$

for periods $t > T$ and conventionally we set $A_{0t}(T) = 0$ if $t \leq T$.

We can now define $c_t^{RoT}(T)$, the daughter's consumption at date t, under the RoT contract. The income is contingent on T, since the mother passes away at the end of this period. We have, for all t,

$$c_{1t}^{RoT}(T) = y_{1t} + (A_1 - A)H_1 + A_{0t}(T)H_0,$$

where $A_{0t} = 0$ for $t \leq T$.

d) In each period $t \leq T$, the mother consumes $c_{0t} = y_0$. After the mother's death ($t > T$), we simply set $c_{0t}(T) = 0$.

e) Finally, we apply the rule of thumb to determine H_1. That is, we choose a fraction ϕ in the interval $[0, 1]$ and we set H_1 so that $AH_1 = \phi y_1$.

6.2 Specification and Calibration

We need to specify the utility functions u and v_1 and v_0. The crucial functions are in fact only u and v_1, since we explore solutions in which the mother's house is sold only when the mother's passes away. We choose standard CRRA utilities, namely,

$$u(c) = \frac{c^{1-\sigma} - 1}{1-\sigma},$$

if $\sigma \neq 1$, where the index of relative risk aversion is $\sigma \geq 0$, and

$$v_1(H) = \frac{H^{1-\eta} - 1}{1-\eta},$$

if $\eta \neq 1$, where $1/\eta$ determines the daughter's elasticity of housing demand. If $\sigma = 1$ we take the limit function $u(c) = \ln(c)$. Similarly, if $\eta = 1$ we take $v_1(H) = \ln(H)$.

In addition, we assume that house prices are predictable with a rate of growth κ, that is, $q_t = q_1(1+\kappa)^{t-1}$.

The key parameters of the model are $(\beta_0, \beta_1, \sigma, \eta, \gamma, \kappa, r)$, the psychological discount factors, the degree of risk aversion, the inverse elasticity of housing demand, the degree of altruism of the mother, the (real) rate of growth of house prices and the base rate of interest.

To calibrate the model completely, we need to specify the joint probability distribution of income and housing assets, that is the joint distribution of (H_0, y_0, y_1). We assume that these variables are jointly log-normal, *i.e.*, that $(\ln(H_0), \ln(y_0), \ln(y_1))$ is normally distributed. To determine the covariance of $\ln(y_0)$ and $\ln(y_1)$, we use the literature on intergenerational transmission of earnings and wealth. Lefranc and Trannoy (2005) have estimated the relation between the daughter's and the mother's income with the help of French data. The standard approach to these questions is to specify a simple econometric model of the form,

$$\ln(y_1) = a_0 + a \ln(y_0) + \epsilon,$$

where ϵ is an independent normal error term. If $\ln(y_1)$ and $\ln(y_0)$ have the same variance, then, the regression coefficient a is equal to the correlation coefficient of the two logarithms. According to Lefranc and Trannoy (2005), a is somewhere between 0.25 and 0.45, closer to 0.4 in the recent years. We choose $a = 0.4$, and equal variances for the log-incomes. The variance of $\ln(y_1)$ is taken from INSEE's *Enquête Patrimoine* and we find that the standard deviation of log-income is 0.83. We assume $Var(\ln(y_1)) = Var(\ln(y_0)) = 0.83^2 = 0.69$. From this we derive,

$$Var(\epsilon) = Var(\ln(y_1))(1 - a^2) = 0.69 \times 0.84 \simeq 0.58.$$

The standard deviation of ϵ is therefore $\sqrt{0.58} = 0.76$. From *Enquête Patrimoine* we also get $\mathbb{E}(\ln(y_0)) = 11$. We assume $\mathbb{E}(\ln(y_0)) = \mathbb{E}(\ln(y_1))$ for simplicity (this corresponds to a stationary environment). This assumption pins down a_0, we have

$$a_0 = (1 - a)\mathbb{E}(\ln(y_1)) \simeq 6.6.$$

We specify the distribution of H_0 in the same way, using French data from INSEE's *Enquête Patrimoine*. We use a regression of the form

$$\ln(H_0) = b_0 + b\ln(y_0) + \xi,$$

where ξ is an independent normal random error term, to model the correlation of H_0 and y_0. The regression results obtained are precise. We obtain the following estimates, $b = 0.25$, $b_0 = 1.88$ and the standard deviation of ξ is equal to 0.39.

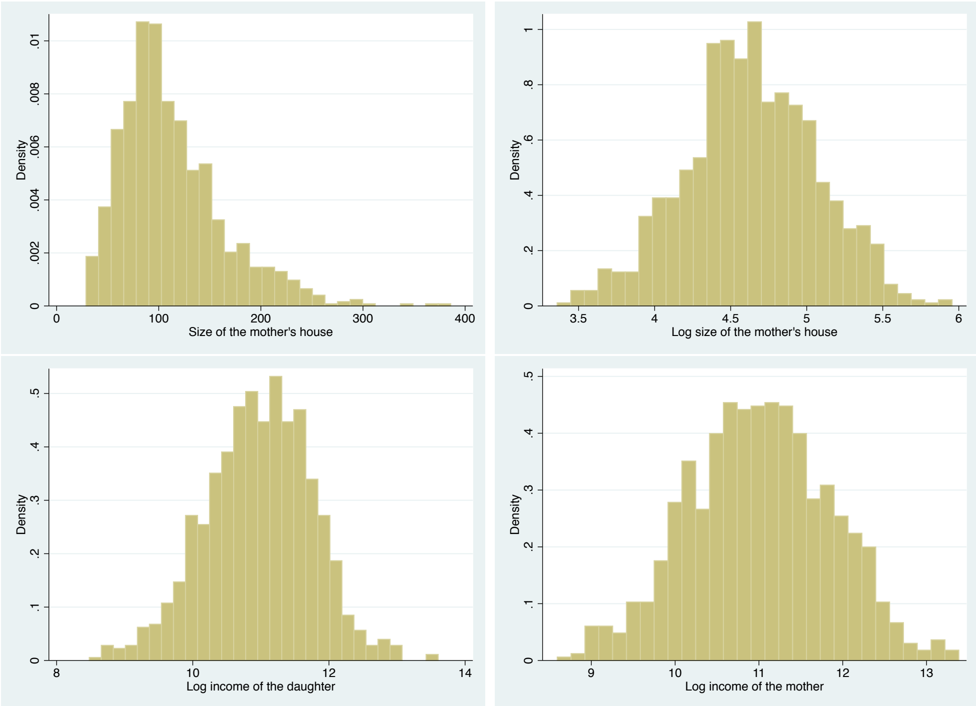
Given these specifications, we draw a random sample of 1000 families in the joint log-normal distribution so obtained, where the variances of $\ln(H_0)$ and $\ln(y_0)$ are chosen in accordance with data sources.

There are $N = 1000$ families indexed by $i = 1, \ldots, N$, and we draw different ages above 50 for the mother at time $t = 0$. The mother's age is drawn uniformly between 50 and 70. From this we derive the maximal number of years of mother i and of daughter i, assuming that all mothers pass away before the age of 100. We also use the appropriate INED life tables to compute the survival function S_T^i and the probability p_T^i for each mother i in the sample.

We can now treat the simulated data by means of a statistical software as if it was a sample of real observations, and we can generate as many samples as we wish by varying the values of the model's structural parameters, to perform sensitivity analysis.

Given this sample of families, we solve the model, for each of the 1000 families in the sample, in 10 different scenarios. These scenarios are different combinations of key parameters. We study the effect of changing three parameters only: risk aversion, altruism and time preference. There is a 'central scenario' based on the choice $(\sigma, \gamma, \beta_0) = (.8, 1, .96)$ and we take $r = .03$, $\rho = .04$, $q_1 = 5250$ (in euros per square meter) with a real rate of growth of 1 percent per year, $\beta_1 = \beta_0$, and $\eta = 0.9$. We then construct the other scenarios by changing the values of each parameter separately while the others are fixed at their central values. The values of σ are chosen in the set $\{.5, .6, .7, .8, .9\}$; the values of γ vary in the set $\{1, 1.5, 2\}$, and the values of β_0 are chosen in the set $\{.95, .96, .97, .98\}$. Figure 20 shows the empirical distributions of the key exogenous

Figure 20: Distribution of log-income for mother and daughter, and distribution of mother's house size



Source: Simulated data set. Key exogenous variables

variables in the simulated sample. Table 18 gives the summary statistics. The figures show that simulated observations represent relatively well-to-do individuals from the Paris region. Real estate prices q_1 are chosen as an average of the Paris region (average prices in the center of Paris would be closer to 8000 euros per square meter). Given the basic data describing each randomly drawn family, we compute the optimal, full-insurance contract as described in sub-section 5.1 above. We compute the numerical values of utilities for each family member and each family i, that is, $U_i(\pi)$, $V_i(\pi)$, where $\pi \geq 0$ is the bank's expected profit from the contract. The utility levels depend on the profit cashed in by the bank on each family: π_i is a summary in present actuarial value terms of the surplus extracted by the bank from family i. The appropriate value of π_i depends on family characteristics. Let π_i^* denote the maximal profit of the bank with the

Table 18: Summary statistics of simulated data

	Mean	Median	Standard deviation
Income of the mother	85,759	59,586	81,440
Income of the daughter	78,143	60,193	69,222
Size of the mother's house	113	102	51

Simulated data set. Yearly incomes are in euros. Areas are in square meters.

Table 19: Summary statistics of simulated profits

	Mean	Median	Standard deviation
$\hat{\pi}(\rho)$ (RoT)	63,545	48,554	59,467
π^* (CwP)	387,320	255,000	454,574

Simulated data set, central scenario. Profits are in euros. RoT refers to rule-of-thumb contract. CwP refers to the optimal contract with maximal profit.

optimal contract. By definition, π_i^* is such that the family is indifferent between the optimal and the RoT contracts. Let U_{i0}, V_{i0} and $W_{i0} = U_{i0} + \gamma V_{i0}$ denote the utility levels of the mother, the daughter and the family, respectively, under the RoT contract. Then, by definition, π_i^* is the value of profit that solves the equation,

$$U_i(\pi_i^*) + \gamma V_i(\pi_i^*) = W_{i0}. \tag{41}$$

Due to the monotonicity properties of indirect utility, this solution is the maximal value of profit generated by a contract with family i. Our simulation program computes this value for each family in the sample. We end up with a data set including exogenous data (H_{i0}, y_{i0}, y_{i1}), and endogenous data

$$(H_{i1}, U_i(\pi_i^*), V_i(\pi_i^*), U_{i0}, V_{i0}, \hat{H}_{i1}, \pi_i^*, \hat{\pi}_i(\rho))$$

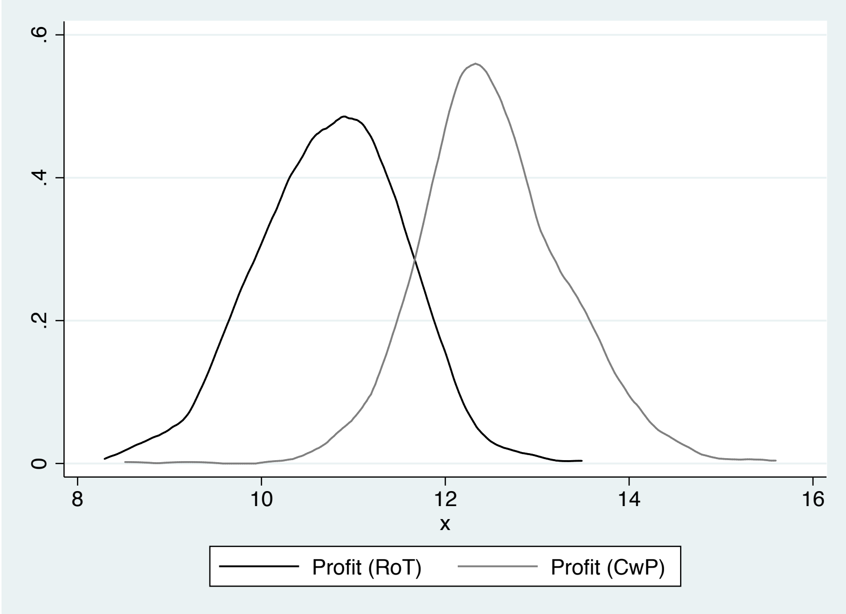
where $\hat{H}_{i1}$ is the daughter's house size and $\hat{\pi}_i(\rho)$ is the profit of the bank under the RoT contract. The latter value of profit is simply,

$$\hat{\pi}_i(\rho) = \phi y_{i1} \sum_{t=1}^{t=\mathcal{T}+1} \beta^{t-1} - q_1 \hat{H}_{i1} = \phi y_{i1} \sum_{t=1}^{t=\mathcal{T}+1} (\beta^{t-1} - \delta^{t-1}) > 0. \tag{42}$$

In particular, an output of interest is the distribution of expected profits π_i^* generated by the profit-maximizing family contracts. Table 19 gives the main descriptive statistics of the simulated profits. Profit is nonnegative and substantial. The reader must keep in mind that the profit, expressed in euros, is the total discounted sum of intertemporal expected benefits, earned by the bank over the entire life of a contract with one family, and that the chosen families are (relatively) rich Parisians. Figure 21 shows the distribution of $\ln(\pi_i^*)$ and $\ln(\hat{\pi}_i(\rho))$. It is clear from Table 19 and Fig. 21 that the bank's profit increases very significantly as compared to the RoT-contract profit. The RoT (*i.e.*, Rule of thumb) and CwP (*i.e.*, Contract with maximal profit) cases can be interpreted as providing a lower and an upper bound on the profit of the bank per capita. Surplus sharing may lead, according to these simulations, to a fourfold or fivefold increase of expected profits per family.

To check how the model works, we ran the artificial OLS regression of log-profit on mother's and daughter's log-income and log-size of the mother's house. Results are given in Table 20. The first column shows that the RoT profit doesn't depend on the mother's wealth and income. In contrast, all covariates become highly significant under the optimal contract, because profit then depends on the mother's income and wealth.

Figure 21: Density of simulated log-profits with RoT and optimal contracts



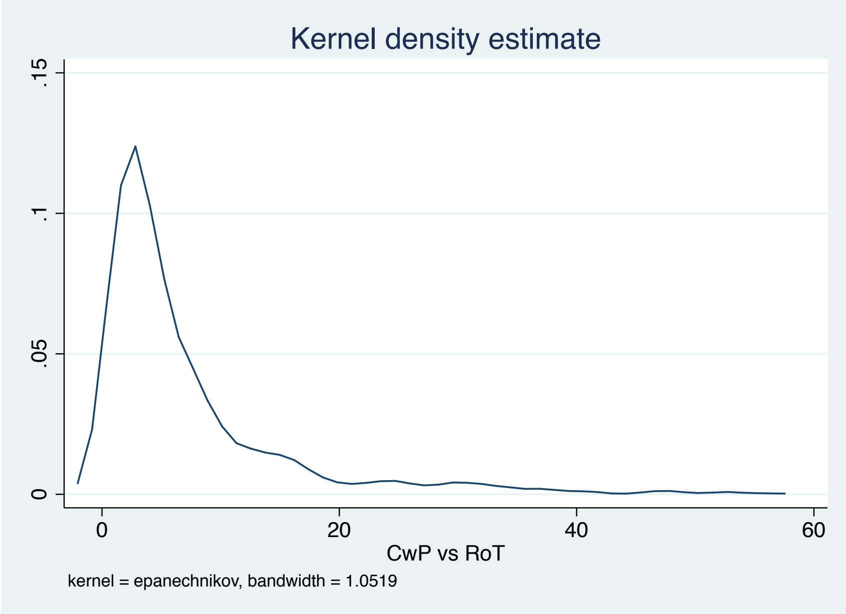
Source: Simulated data set, central scenario. RoT refers to Rule of thumb contract. CwP ('contract with profit') refers to the optimal contract with maximal profit π^*.

Table 20: Regression of log-profit on log-incomes and log-size of the mother's house

	Log-Profit (RoT contract)	Log-Profit (Optimal contract)
Mother's Log income	-0.001	0.158***
	(0.001)	(0.031)
Log-size of mother's house	0.000	0.535***
	(0.001)	(0.055)
Daughter's Log-income	1.001***	0.151***
	(0.001)	(0.030)
Mother's Age	-0.030***	-0.048***
	(0.000)	(0.004)
Constant	1.610***	9.563***
	(0.010)	(0.403)
R^2	1.00	0.34
Number of observations	1,000	1,000

Simulated data set, central scenario. Method: OLS

Figure 22: Density of relative variations of profits between RoT and optimal contracts



Source: Simulated data set, central scenario. RoT refers to Rule of thumb contract. CwP ('contract with profit') refers to the optimal contract with maximal profit π^*.

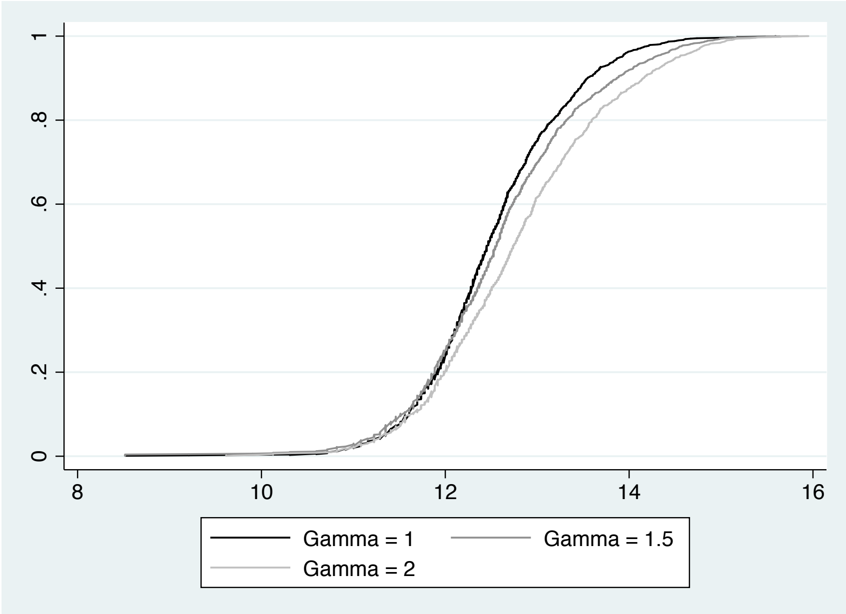
The next step is to look at the distribution of $(\pi_i^* - \pi_i(\rho))/\pi_i(\rho)$, the relative variations of profit. This distribution is presented in Fig. 22. The figure shows that profit can be multiplied by more than 3 or 4 (or even more) as compared to the RoT contract. The relative profit increase may be particularly large in some relatively less likely cases in which the mother is rich and the daughter has a small income.

We now study the impact of some key parameters and compare profits in different scenarios. First we look at the effect of increasing the mother's degree of altruism. Fig. 23 gives the CDF of $\ln \pi^*$ in the sample for the three values of γ: 1, 1.5 and 2. We see that the distributions obtained with a higher value of the altruism parameter γ (almost) dominate the others in the sense of first-order stochastic dominance (hereafter FOSD). Recall that distribution F dominates distribution G in the sense of first-order stochastic dominance if the cdf of F is everywhere smaller or equal than the cdf of distribution G. This confirms that a more altruist family is more profitable for the bank.

If we now decrease the discount factor β_0, do we observe that the new distribution of π^* dominates the former in the sense of FOSD? The answer is yes: a less patient family (a lower β_0) is more profitable for the bank, as shown by Figure 24. Recall that $\beta_0 = \beta_1$ here. The FOSD ranking is particularly obvious in the case of Fig. 24.

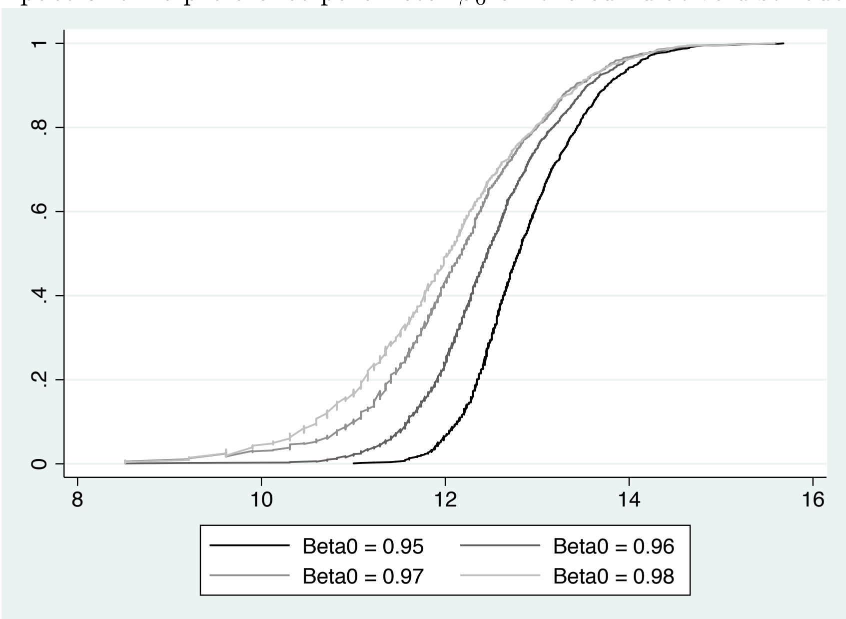
Finally, we computed the model's solutions for several values of the CRRA risk-aversion parameter σ. The values are .5,.6,.7,.8 and .9. Figure 25 plots the numerical results. There is again a clear ranking of distributions. To a certain extent, a more risk-averse family is more profitable, since the distribution for $\sigma = .9$ dominates the others. But the effect of risk aversion

Figure 23: Impact of altruism parameter γ on the cumulative distribution of profits



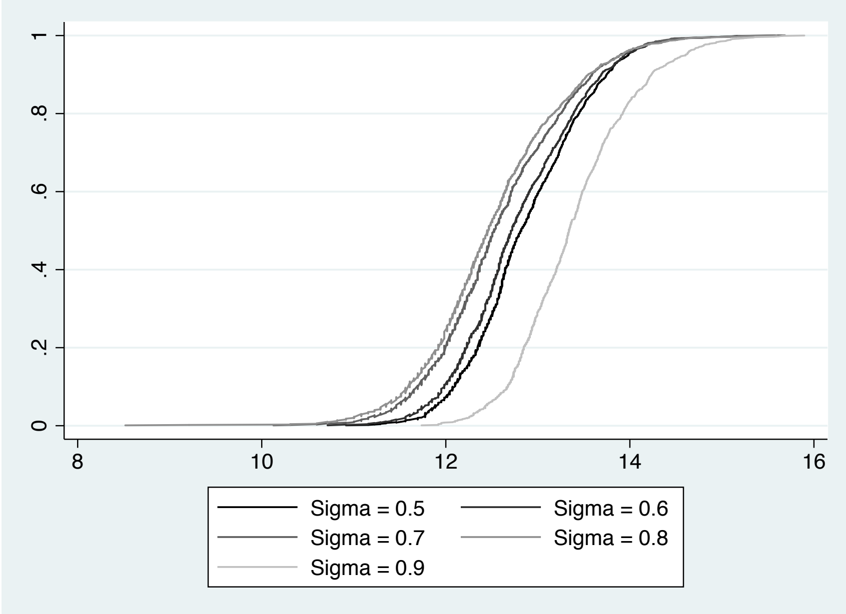
Source: Simulated data set, γ takes the three values: 1, 1.5 and 2. The figure represents the cumulative distribution function of log-profits, $\ln(\pi^*)$ for the three values of γ. First-order stochastic dominance of distribution F over G is characterized by the property that the cdf of F is everywhere smaller than the cdf of distribution G.

Figure 24: Impact of time preference parameter β_0 on the cumulative distribution of profits



Source: Simulated data set, β_0 takes the four values: .95, .96, .97 and .98. The figure represents the cumulative distribution function of log-profits, $\ln(\pi^*)$ for the four values of β_0.

Figure 25: Impact of risk-aversion parameter σ on the cumulative distribution of profits



Source: Simulated data set, σ takes the 5 values: .5,.6,.7,.8 and .9. The figure represents the cumulative distribution function of log-profits, $\ln(\pi^*)$ for the 5 values of σ.

is highly nonlinear here.

Our next step is to study the distribution of the daughter's house size H_1, under the rule of thumb contract (RoT), the Optimal Contract with zero profit (Cw0P), and the optimal contract with maximal profit (CwP). Table 21 gives the summary statistics of the simulated distribution of H_1 under the three contracts. The optimal contract, with zero or maximal profit, permits a substantial increase in the daughter's house value. Figure 26 permits one to compare the mother's and the daughter's house sizes. It is easy to see that under the optimal contract (with or without profit), the distributions of H_0 and H_1 are much closer than under the RoT contract. It is also interesting to see the distribution of relative variations in the daughter's house size between different contractual arrangements. The distribution of percentage variations $\Delta H_1/H_1$ are given in Figure 27. Let $\hat{H}_1(\rho)$ denote the daughter's house area under the RoT contract, let H_1^* denote the daughter's house area under the optimal contract with maximal profit (CwP); let finally H_1^0 denote the daughter's house size under the optimal contract with zero profit. The top plot in Figure 27 gives the three smoothed densities of $(H_1^0 - H_1^*)/H_1^*$, $(H_1^* - \hat{H}_1(\rho))/\hat{H}_1(\rho)$ and $(H_1^0 - \hat{H}_1(\rho))/\hat{H}_1(\rho)$. The first of these distributions is very concentrated as compared to the other two, looking like a spike on the top plot. This shows that the optimal contract with profit and with zero profits are in fact very close in terms of the daughter's house size. The other two distributions are the relative variations of house size under the optimal contract, with respect to the RoT contract. The latter distributions are much more dispersed.

The middle picture in Fig. 27 shows that the bulk of the percentage variation between optimal and RoT contracts is between 0% and 200%. There are huge variations, mainly due to the fact

Table 21: Size of the daughter's house

	Mean	Median	Standard deviation
Size of daughter's house (RoT)	76	58	69
Size of daughter's house (CwoP)	107	91	68
Size of daughter's house (CwP)	97	82	63

Simulated data set, central scenario. House size in square meters.

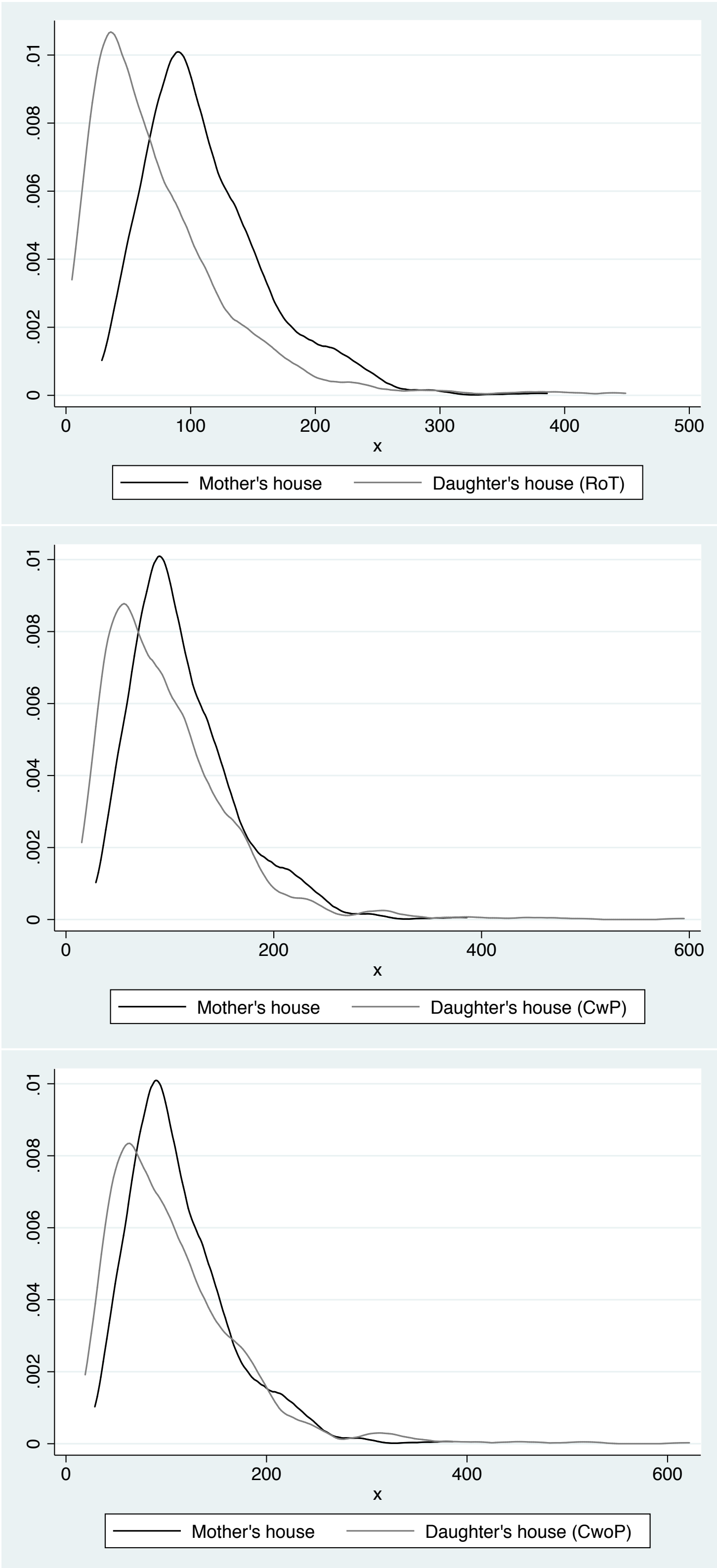
that the mother's wealth is now transmitted to the daughter sooner than when it takes the form of inheritance, and is also taken into account to assess the mortgage repayment ability of the daughter. The bottom plot on Fig. 27 shows that most variations in real-estate investment, due to the fact that profit would vary from its maximal value to zero, are in the range of 0 to 25%.

As a check, we run the regression of the log-house size on a number of covariates measuring income and wealth. The results are given by Table 22. In these regressions based on artificial data, we see that the mother's income and real estate assets are significant only under the optimal contracts (second and third columns of the table). It is also possible to explore the alternative scenarios to study the effect of key parameters on the daughter's real estate investment. First, we look at the effect of increasing the mother's degree of altruism. Fig. 28 gives the CDF of H_1^* in the sample for the three values of γ: 1, 1.5 and 2. We see that the distributions obtained with a higher value of the altruism parameter γ dominate the others in the sense of first-order stochastic dominance. This confirms that a more altruist mother causes a larger real estate investment on the part of the daughter. If we now increase the discount factor β_0, do we observe that the new distribution of H_1^* dominates the former in the sense of FOSD? The answer is yes: a more patient family (a larger β_0) enables a larger home for the daughter, as shown by Figure 29, but the impact is small.

Finally, we computed the optimal house size for several values of the CRRA risk-aversion parameter σ. The values are .5,.6,.7,.8 and .9. Figure 30 plots the numerical results, showing cumulative distributions of H_1^*. There is again a clear ranking of these distributions. We conclude that a more risk-averse daughter chooses a larger investment in real estate, in view of the clear FOSD ranking.

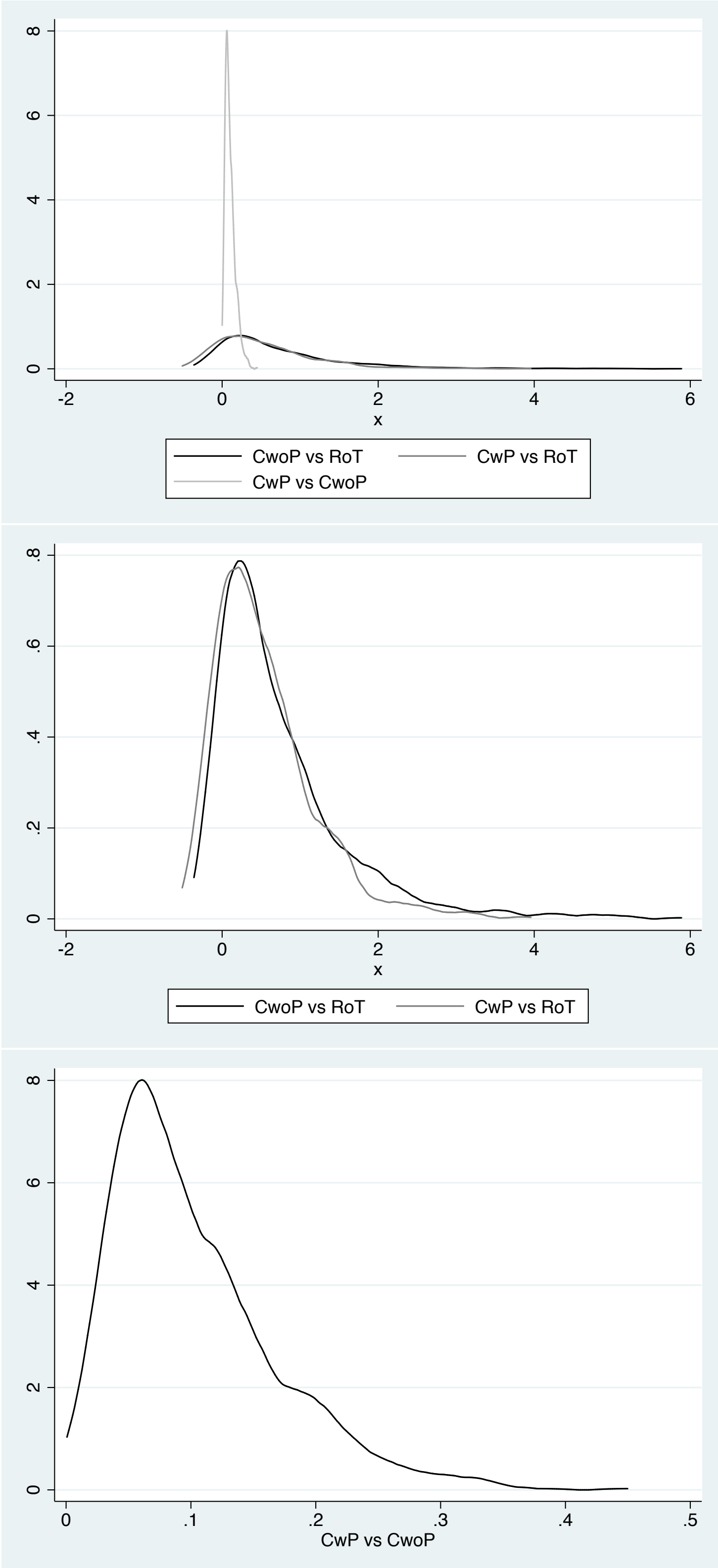
We conclude this discussion of our illustrative simulations with a look at the distribution of the relative variation of welfare $W = U + \gamma V$ in the central scenario. By definition, the variation of welfare between the RoT contract and the optimal contract with maximal profit (CwP) is zero (up to precision problems in the computation algorithm). Hence, we plot the distribution of relative variations of W, in the central scenario, between the RoT contract and the optimal zero-profit contract (Cw0P). This distribution is depicted in Figure 31. It provides indications on the upper bound for welfare improvement attached to the optimal contract. The bulk of these improvements are in the range of 5 to 10%, which is not negligible.

Figure 26: Comparison of the mother's and the daughter's house-size distributions



Source: Simulated data set, central scenario. House size in square meters.

Figure 27: Distribution of percentage variation in daughter's house size



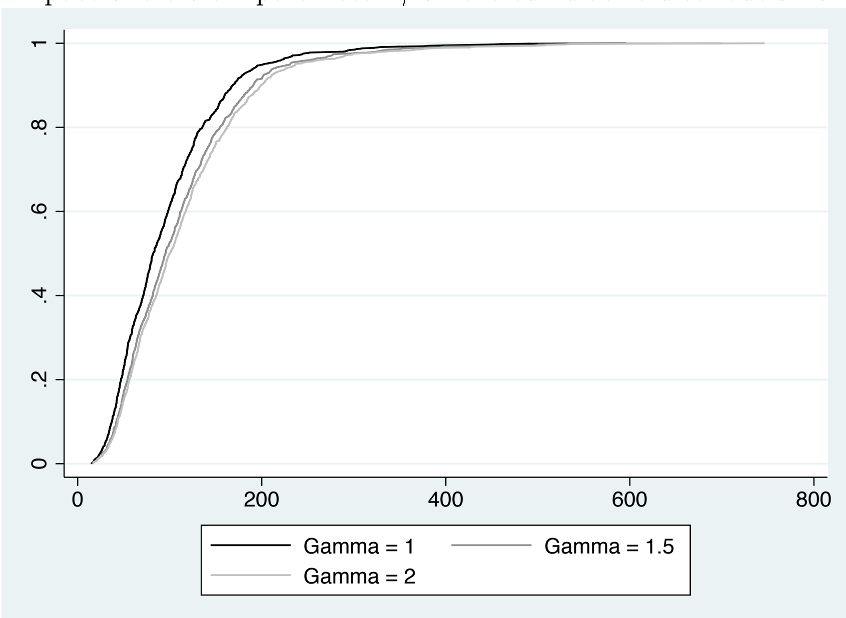
Source: Simulated data set, central scenario. House-size variations in percentage. RoT refers to Rule of thumb contract. Cw0P means optimal contract with zero profit. CwP means optimal contract with maximal profit π^*.

Table 22: Regression of the daughter's log-house-size on log-incomes and log-size of the mother's house

	$\ln H_1$ under RoT	$\ln H_1$ under CwoP	$\ln H_1$ under CwP
Mother's log-income	-0.000	0.357***	0.374***
	(0.000)	(0.004)	(0.001)
Log-size of mother's house	0.000	0.085***	0.042***
	(0.001)	(0.008)	(0.002)
Daughter's log-income	1.001***	0.440***	0.481***
	(0.000)	(0.004)	(0.001)
Age of the mother	-0.010***	0.003***	0.006***
	(0.000)	(0.001)	(0.000)
Constant	-6.314***	-4.845***	-5.525***
	(0.004)	(0.055)	(0.015)
R^2	1.00	0.98	1.00
Number of observations	1,000	1,000	1,000

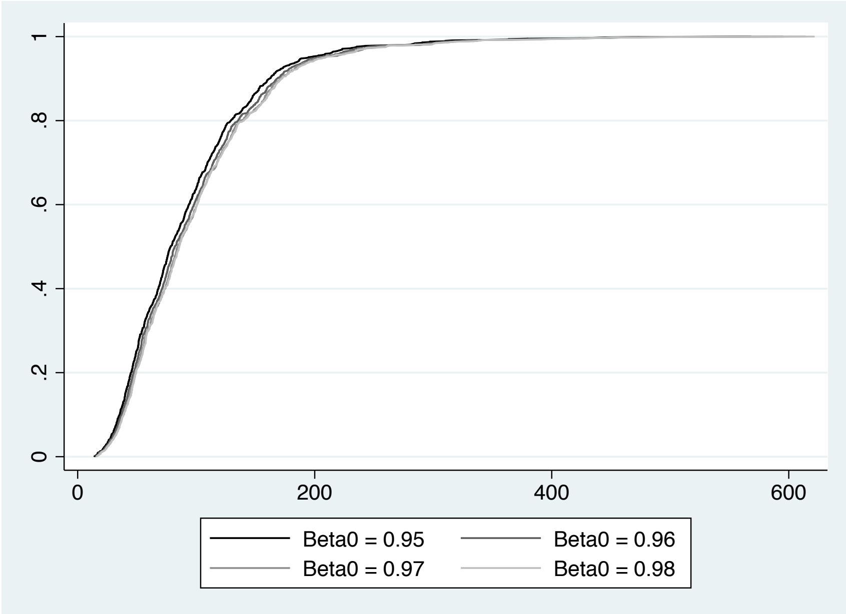
Simulated data set, central scenario. Method: OLS

Figure 28: Impact of altruism parameter γ on the cumulative distribution of house area



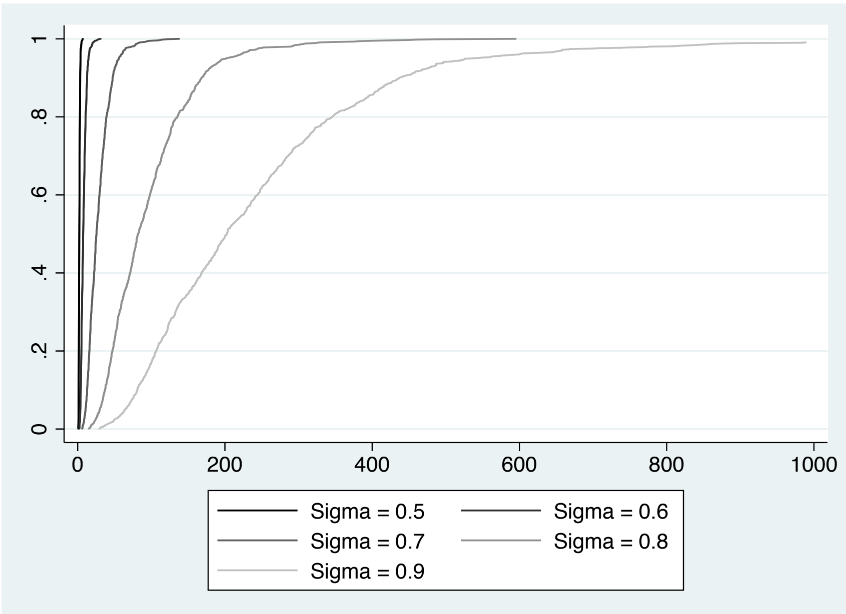
Source: Simulated data set, γ takes the three values: 1, 1.5 and 2. The figure represents the cumulative distribution function of the daughter's house area in square meters, H_1^* for the three values of γ. First-order stochastic dominance of distribution F over G is characterized by the property that the cdf of F is everywhere smaller than the cdf of distribution G.

Figure 29: Impact of time preference parameter β_0 on the cumulative distribution of the daughter's house size



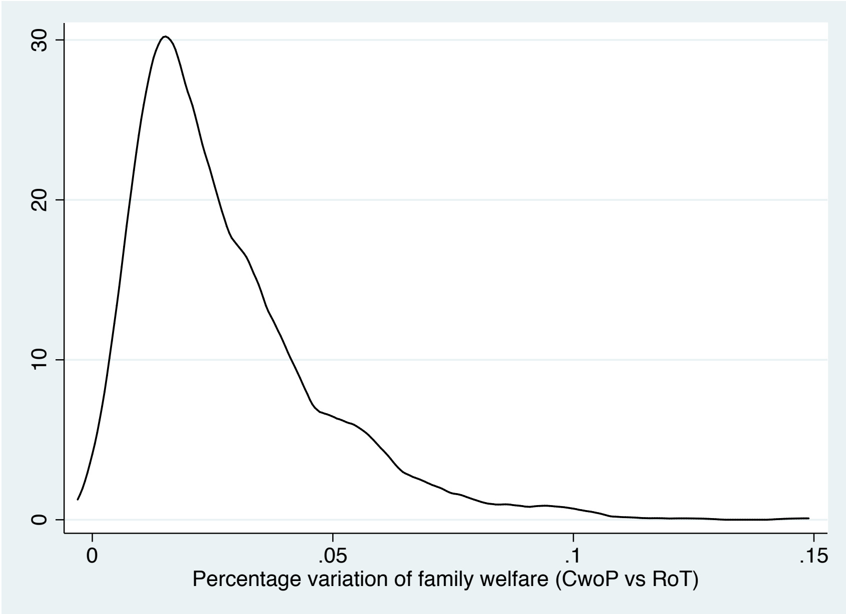
Source: Simulated data set, β_0 takes the four values: .95, .96, .97 and .98. The figure represents the cumulative distribution function of the daughter's house, H_1^* for the four values of β_0.

Figure 30: Impact of risk-aversion parameter σ on the cumulative distribution of the size of the daughter's house



Source: Simulated data set, σ takes the 5 values: .5,.6,.7,.8 and .9. The figure represents the cumulative distribution function of log-house-size, $\ln(H_1^*)$ for the 5 values of σ.

Figure 31: Distribution of percentage variation in family welfare between rule of thumb and optimal zero-profit contract



Source: Simulated data set, central scenario. The figure represents the smoothed density function of percentage variations in family welfare, $W = U + V$, between Cw0P and RoT contract.

7 Concluding Remarks. Design of Family Contracts, Behavioral Obstacles and Marketing

The above theory and the associated numerical analysis, based on a fully specified and calibrated version of our model, both suggest that there is room for improvements in the efficiency of family arrangements, relative to standard practices of inter-vivos gifts and inheritance. Our contribution is not to emphasize the importance of equity release instruments for the wealth management of the elderly. More precisely, our contribution is to study the combination of equity release contracts with more general insurance and household investment strategies, involving several members of a family and different generations. A banker and an insurer can jointly improve the efficiency of family arrangements, in particular, because they can radically improve risk-sharing within the family. The banker-insurer can immunize the family against real-estate price risk and at the same time immunize the heirs (children or spouse, or both) against longevity risk, by means of structured inter-temporal transfers between parents and children. The banker-insurer acts as a middleman for family members. The gifts to children being in fact carried out by the financial intermediary, they can be made independent of the event of a parent's death. Under the terms of a family contract, the banker-insurer supplies the family with a bundle of services: liquidity, various kinds of loans and insurance; the financial intermediary satisfies the demand for bequest and gifts, due to the altruist motives of the parents, on behalf of the parents. The intermediated family arrangement can be interpreted as a bundle of bilateral contracts with the bank.

Another important question is the possibility of removing the behavioral obstacles to the use of equity-release instruments and annuitization of wealth. If the elderly have a bequest motive (based on pure altruism or some taste for giving), they could annuitize their wealth partially, and ring-fence part of their assets for inheritance. But if the children are just slightly impatient, then the money should be transmitted as soon as possible in the form of an inter-vivos gift. This could happen, say, at the latest of the two moments when the parents have completely repayed the mortgage on their residence and when their children reach adulthood and want to settle. In practice, given the facts studied in Section 2 above, the elderly members of the family may in fact wish to help their grandchildren, because they are typically still young when their own children want to buy a house. Home reversion plans or intermediated viager contracts can do the job of transmitting the present value of the parent's real estate assets sooner than the death of the last surviving member of the parent's household. It's rational to annuitize the real-estate assets by means of a viager or home reversion contract only if the family seniors are at the same time well insured against the need for long-term care, *i.e.*, the risk of moving to a nursing home or the risk of a sharp increase in health expenditures. Our model should and could be extended to take into account the long-term care and health risks, at the cost of additional complexity. But it's clear that the banker-insurer can price the relevant risks with a reasonable degree of accuracy and solve the equity-release and annuitization problems completely. What remains is to structure the timing and sharing of the estate among family members, and in so doing, to insure the heirs against the father's (or the mother's) longevity risk. The banker-insurer can do something that the mother or the father could not do without his help by just selling his (her) house on a viager or home reversion market. On her own, the mother could share the annuity value of the *viager* sale of her house among her children. This move would insure the whole family against future house-price risk. But the money distribution, *i.e.*, transfers to children would stop when the mother passes away. This is a risk if the children still have to repay mortgages at this moment. The banker-insurer can commit to continue subsidizing the children's asset building efforts during a time span that is completely independent of the mother's longevity. This is the key element in our theory of intermediated family contracts. There are of course some tax optimization aspects in the design of these contracts. Since the intermediated family arrangement can be interpreted as a bundle of bilateral contracts with the bank, it is unclear whether the transfers of purchasing power between family members implemented by the bundle of contractual arrangements can be interpreted (in legal terms) as gifts, or if they must be viewed as cross-subsidies between several categories of the bank's clients, or even simply as promotional pricing aimed at new customers. It may be that inter-vivos gifts are taxed, in a given country, while cross-subsidies between clients or products are in fact a legally admissible form of (third-degree) price discrimination.

Assuming that the long-term care insurance problem has been solved, it is reasonable to expect that an intermediated family contract would help removing the behavioral obstacles to annuitization of the elderly's wealth, because annuitization becomes part of an encompassing solution to a family problem. Of course, further empirical research would be needed to really estimate the extent to which the latter claim is true.

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9 Appendix: Proofs

Proof of Proposition 1. Let λ be the Lagrange multiplier of the actuarial resource constraint (4) above; let ν_t be the multiplier of $Z_t \geq 0$; let μ be the multiplier of constraint

$$H_0 - z_1 - z_2 - z_0 \geq 0;$$

Taking derivatives with respect to z_0 first, assuming interiority, *i.e.*, $z_0^* > 0$, we find the necessary conditions

$$\lambda X \tau_b = \mu; \tag{43}$$
$$\mu(H_0 - z_0 - z_1 - z_2) = 0; \tag{44}$$
$$\lambda \geq 0; \qquad \mu \geq 0; \tag{45}$$

It follows immediately that if $\lambda > 0$ and $\tau_b > 0$ then $\mu > 0$ and $H_0 = z_0 + z_1 + z_2$. But these conditions do not pin down z_0^*.

To do this, we write the first order conditions with respect to z_1 and z_2. This yields,

$$-v'_{0h}(H_0 - z_1, H_0) - \beta_0 p v'_{0h}(H_0 - z_1 - z_2, H_0) = \lambda[(1-\tau_b)X - q_1] - \nu_1 + \mu; \tag{46}$$
$$-\beta_0 p v'_{0h}(H_0 - z_1 - z_2, H_0) - \lambda(1-\tau_b)X = -\lambda \frac{q_2}{(1+r)}(1 - \tau_b(1-p)) - \nu_2 + \mu; \tag{47}$$
$$\nu_1 z_1 = \nu_2 z_2 = 0; \tag{48}$$
$$\mu(H_0 - z_0 - z_1 - z_2) = 0; \tag{49}$$
$$(\mu, \nu_1, \nu_2) \geq 0. \tag{50}$$

But using $\lambda X \tau_b = \mu$ we find,

$$(1 + \beta_0 p) v'_{0h}(H_0, H_0) + \lambda(X - q_1) = \nu_1 > 0; \tag{51}$$
$$\beta_0 p v'_{0h}(H_0, H_0) + \lambda \left[X - \frac{q_2}{(1+r)}(1 - \tau_b(1-p)) \right] = \nu_2 > 0. \tag{52}$$

These inequalities will always be true if the left derivative of v_0 at point (H_0, H_0) is large enough. It follows, that $z_1^* = z_2^* = 0$. It is easy to check (see below) that $\lambda^* > 0$. As a consequence, since $\tau_b > 0$ by assumption, we have $H_0 = z_0^*$. *Q.E.D.*

Proof of Proposition 2. We first take derivatives with respect to H_1, assuming interiority, that is, $H_1^* > 0$. We obtain the following first-order condition.

$$\begin{aligned} &v'_1(H_1 + h_1) + p\beta_1 v'_1(H_1 + h_2) + p\beta_1^2 v'_1(H_1 + h_3) \\ &\qquad + (1-p)\Big(\beta_1 v'_1(H_1 + \hat{h}_2) + \beta_1^2 v'_1(H_1 + \hat{h}_3)\Big) \\ &\qquad = \frac{\lambda}{\gamma}\left(q_1 - \frac{q_4}{(1+r)^3}\right). \end{aligned} \tag{53}$$

If we take derivatives with respect to h_t and $\hat{h}_t$, we get the following conditions

$$\gamma v_1'(H_1 + h_1) \leq \lambda R_1; \quad h_1 \geq 0 \tag{54}$$

$$\gamma \beta_1 v_1'(H_1 + h_2) \leq \lambda \frac{R_2}{(1+r)}; \quad h_2 \geq 0 \tag{55}$$

$$\gamma \beta_1^2 v_1'(H_1 + h_3) \leq \lambda \frac{R_3}{(1+r)^2}; \quad h_3 \geq 0 \tag{56}$$

$$\gamma \beta_1 v_1'(H_1 + \hat{h}_2) \leq \lambda \frac{R_2}{(1+r)}; \quad \hat{h}_2 \geq 0 \tag{57}$$

$$\gamma \beta_1 v_1'(H_1 + \hat{h}_3) \leq \lambda \frac{R_3}{(1+r)^2}; \quad \hat{h}_3 \geq 0. \tag{58}$$

where the inequalities become equalities if the corresponding $h_t^* > 0$ or $\hat{h}_t^* > 0$. Substituting the above inequalities in the first-order condition for an optimal H_1 above, we find the arbitrage inequality given in the statement of Proposition 2 above. When $h_t^* > 0$ and $\hat{h}_t^* > 0$, the above inequalities become equalities and the arbitrage equation holds exactly. If $h_t^* = 0$ and $\hat{h}_t^* = 0$, the first-order condition for H_1^* boils down to the expression given as point 3 in the statement of Proposition 2. *Q.E.D.*

Proof of Proposition 3. The conditions stated in Proposition 3 above are straightforward consequences of the first-order optimality conditions, applied to consumption. *Q.E.D.*

Proof of Proposition 4. To prove this proposition, let α_t be the Lagrange multiplier of the sign constraint $x_t \geq 0$. The Kuhn-Tucker conditions for an optimal gift x_t^* can be written as follows. We have $\alpha_t \geq 0$, and $\alpha_t x_t = 0$, together with the conditions,

$$\alpha_1 = \lambda \tau_g; \tag{59}$$

$$\alpha_2 = \frac{\lambda p \tau_g}{(1+r)}. \tag{60}$$

It follows from this, that, if $\tau_g > 0$, then it must be that $\alpha_t^* > 0$, implying $x_t^* = 0$. *Q.E.D.*

Proof of Proposition 6. To prove this proposition we write the Kuhn and Tucker necessary conditions for optimality (maximizing the mother's expected utility subject the ARC 38. We introduce Lagrange multipliers, denoted ν_i, $i = 0, \ldots, \mathcal{T}$ for the z_i variables and the complementary slackness conditions $z_i \geq 0$, $\nu_i z_i = 0$. We introduce a Lagrange multiplier μ for the constraint $\Sigma_0^{\mathcal{T}} z_t \leq H_0$ and we have the condition $\mu(H_0 - \Sigma_0^{\mathcal{T}} z_t) = 0$. Let finally λ be the Lagrange multiplier of the actuarial resource constraint 38. The first-order condition for z_0, assuming $z_0 > 0$, is the following,

$$\lambda X = \lambda \sum_{T=1}^{\mathcal{T}} \beta^T (1 - \tau_b) p_T q_{T+1} + \mu.$$

Using the expression for X, this can be rewritten,

$$\lambda \tau_b \sum_{T=1}^{\mathcal{T}} \beta^T p_T q_{T+1} = \mu. \tag{61}$$

From this equation, we derive that $\lambda > 0$ implies $\mu > 0$, and therefore, $H_0 = \Sigma_{t=0}^{\mathcal{T}} z_t$ (the mother's house is entirely sold before death).

We now write the first-order conditions for the optimality of $(z_1, \ldots, z_{\mathcal{T}})$. For $t > 0$ we find,

$$\sum_{\tau=1}^{\mathcal{T}} \beta_0^{\tau-1} S_\tau v_0' \left(H_0 - \sum_{\theta=1}^{\tau} z_\theta \right) \left(-\frac{\partial \sum_{\theta=1}^{\tau} z_\theta}{\partial z_t} \right)$$

$$+\nu_t + \lambda \beta^{t-1} S_t q_t - \lambda \sum_{T=1}^{\mathcal{T}} p_T \beta^T q_{T+1} (1 - \tau_b) \left(\frac{\partial \sum_{\theta=0}^{T} z_\theta}{\partial z_t} \right) = \mu.$$

Now, given that

$$\frac{\partial \sum_{\theta=1}^{\tau} z_\theta}{\partial z_t} = \begin{cases} 1 & \text{if} \quad t \leq \tau \\ 0 & \text{if} \quad t > \tau, \end{cases}$$

the first-order condition for (z_t), $t > 0$ can be rewritten

$$\mu = -\sum_{\tau=t}^{\mathcal{T}} \beta_0^{\tau-1} S_\tau v_0' \left(H_0 - \sum_{\theta=1}^{\tau} z_\theta \right) + \nu_t + \lambda \beta^{t-1} S_t q_t - \lambda (1 - \tau_b) \sum_{T=t}^{\mathcal{T}} p_T \beta^T q_{T+1}.$$

This can again be rewritten,

$$\nu_t = \sum_{\tau=t}^{\mathcal{T}} \beta_0^{\tau-1} S_\tau v_0' \left(H_0 - \sum_{\theta=1}^{\tau} z_\theta \right) + \mu + \lambda (1 - \tau_b) \sum_{T=t}^{\mathcal{T}} p_T \beta^T q_{T+1} - \lambda \beta^{t-1} S_t q_t. \tag{62}$$

Given 61, we know that $\mu > 0$. Thus, all the terms on the right hand side of 62 are positive except the last one. If the left-derivative of v_0 at point H_0 is large enough, then, we have $\nu_t > 0$ for all $t > 0$ at the optimum. By complementary slackness conditions, we conclude that $z_t = 0$ at the optimum for all $t > 0$. This in turn implies (with $\mu > 0$) that $z_0 = H_0$ at the optimum: the mother sells her entire house in the form of a viager contract at time $t = 1$.

Next, assuming interiority, that is $H_1 > 0$ at the optimum, we take the derivative of the Lagrangian with respect to H_1 and we find the condition,

$$\sum_{t=1}^{\mathcal{T}+1} \beta_1^{t-1} S_t v_1' (H_1 + h_t) + \sum_{T=1}^{\mathcal{T}} p_T \sum_{t=T+1}^{\mathcal{T}+1} \beta_1^{t-1} v_1' (H_1 + h_t(T)) = \frac{\lambda}{\gamma} (q_1 - q_{\mathcal{T}+2} \beta^{\mathcal{T}+1}). \tag{63}$$

Let now η_t (resp. $\eta_t(T)$) denote the Lagrange multiplier of the sign constraint $h_t \geq 0$ (resp. $h_t(T) \geq 0$). Kuhn and Tucker's Theorem tells us that these multipliers are nonnegative. Taking partial derivatives of the Lagrangian with respect to h_t and $h_t(T)$ and equating them to zero yields the following necessary conditions,

$$\gamma \beta_1^{t-1} S_t v_1' (H_1 + h_t) + \eta_t = \lambda \beta^{t-1} S_t R_t, \tag{64}$$

for all $t = 1, \ldots, \mathcal{T}$, and with the condition $\eta_t h_t = 0$, and,

$$\gamma \beta_1^{t-1} v_1' (H_1 + h_t(T)) + \eta_t(T) = \lambda \beta^{t-1} p_T R_t, \tag{65}$$

for $t > T$ and all $T = 1, \ldots, \mathcal{T}$ with the condition $\eta_t(T) h_t(T) = 0$. Now, we substitute 64 and 65 in 63 and we eliminate the terms involving v_1'. This yields,

$$q_1 \leq q_{\mathcal{T}+2} \beta^{\mathcal{T}+1} + \sum_{t=1}^{\mathcal{T}+1} \beta_1^{t-1} S_t R_t + \sum_{t=2}^{\mathcal{T}+1} \beta_1^{t-1} R_t \sum_{T=1}^{t-1} p_T.$$

using the fact that $S_1 = 1 - P_1 = 1$ and $S_{\mathcal{T}+1} = 0$, we finally obtain the arbitrage inequality,

$$q_1 \leq \sum_{t=1}^{\mathcal{T}+1} \beta^{t-1} R_t + q_{\mathcal{T}+2} \beta^{\mathcal{T}+1}. \tag{66}$$

Inequality 66 becomes an equality if the daughter rents at least part of her housing area, since $h_t > 0$, $h_t(T) > 0$ implies $\eta_t = 0$, $\eta_t(T) = 0$.

If 66 holds, then $h_t = 0$, $h_t(T) = 0$ is the solution, and the daughter's house size H_1 is determined by the following equation, derived from 63. Assuming that the effective price of property $q_1 - q_{\mathcal{T}+2}\beta^{\mathcal{T}+1}$ is positive, we have,

$$v_1'(H_1) = \frac{\lambda}{\gamma} \frac{(1-\beta_1)}{(1-\beta_1^{\mathcal{T}+1})} \left(q_1 - q_{\mathcal{T}+2} \beta^{\mathcal{T}+1} \right). \tag{67}$$

Taking the partial derivatives of the Lagrangian with respect to consumption variables and equating to zero, we find $u'(c_{01}) = \lambda > 0$ since $S_1 = 1$ and

$$\beta_0^{t-1} u'(c_{0t}) = \lambda \beta^{t-1} \tag{68}$$

$$\gamma \beta_1^{t-1} u'(c_{1t}) = \lambda \beta^{t-1} \tag{69}$$

$$\gamma \beta_1^{t-1} u'(c_{1t}(T)) = \lambda \beta^{t-1}. \tag{70}$$

From the latter equations, we easily derive points 1-4 in the statement of Proposition 6 and the full insurance property.

Finally, let α_t be the Lagrange multiplier of the sign constraint on gifts x_t, that is, $x_t \geq 0$, $\alpha_t \geq 0$ and $\alpha_t x_t = 0$. Kuhn-Tucker conditions yield $\alpha_t = \lambda \beta^{t-1} S_t \tau_g$. But $\lambda \tau_g > 0$ implies $\alpha_t > 0$ and therefore $x_t = 0$ at the optimum. *Q.E.D.*